WARMUP!! (NO CALCULATORS!!)

1) Find dy/dx if $3(\ln x)y + 2y^3 = 5$

2) Find the equation of the line tangent to $y = (x^2 - 2)\ln(x^3)$ at the point (1, 0)

3) Find and identify all relative extrema and points of inflection of $y = (3\ln x)/(2x) - 4$

1) $\frac{dy}{dx} = \frac{-y}{x\ln x + 2xy^2}$

✓

2) $y = -3x + 3$

3) rel max @ $(e, 3/(2e) - 4)$ b/c y' changes from + to -

p.o.i. @ $(e^{3/2}, 9/(4e^{3/2}) - 4)$ b/c y'' changes sign

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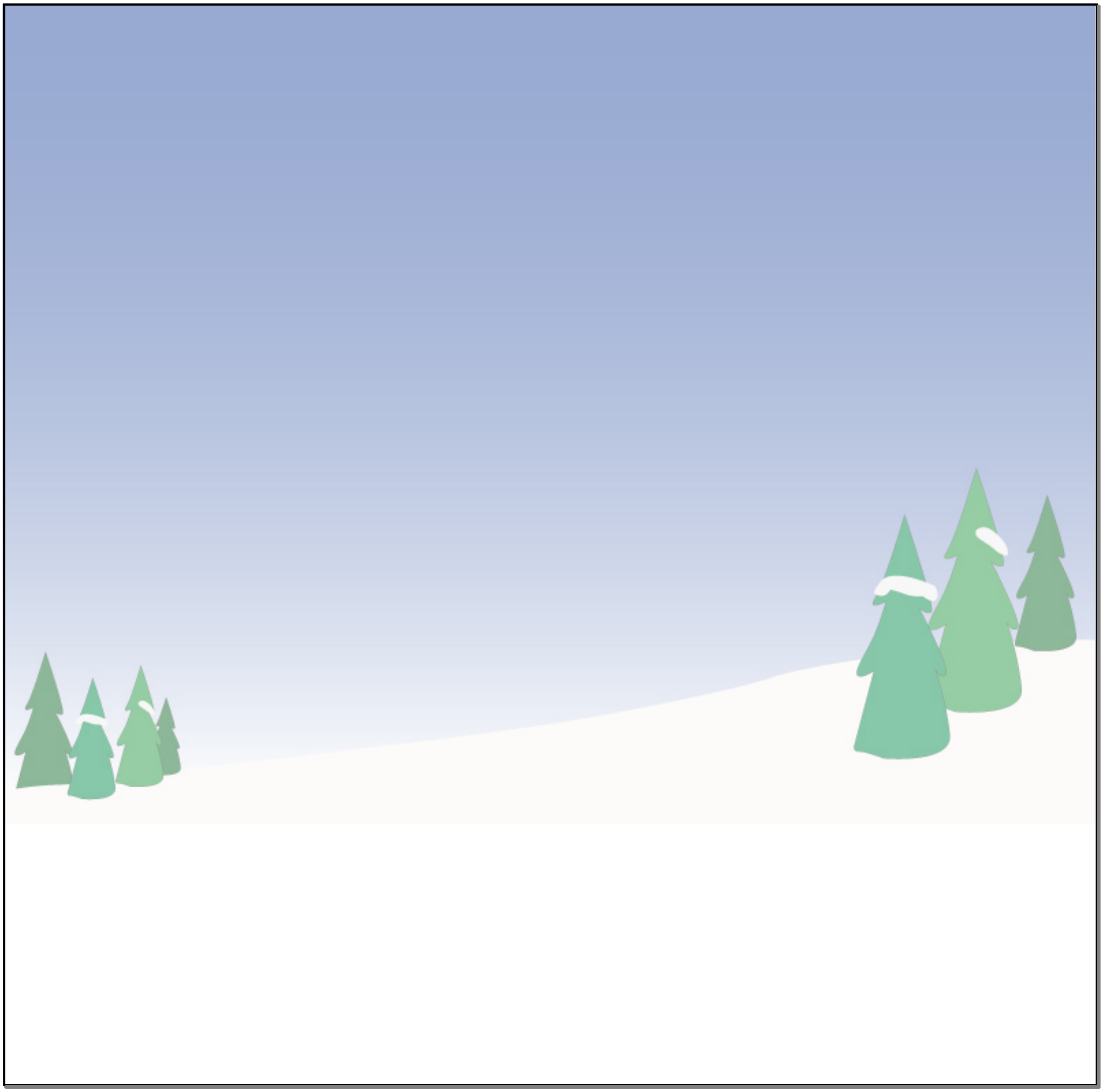

53 $g(t) = \dfrac{\ln t}{t^2}$

$$g'(t) = \frac{\frac{1}{t}\cdot t^2 - \ln t \cdot 2t}{t^4}$$

$$= \frac{t - 2t\ln t}{t^4}$$

$$= \frac{1 - 2\ln t}{t^3}$$

55 $y = \ln(\ln x^2)$

$$y' = \frac{\frac{2x}{x^2}}{\ln x^2} = \frac{2}{x \ln x^2} = \frac{2}{2x \ln x}$$

$$= \frac{1}{x \ln x}$$

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$$y=\frac{-\sqrt{x^2+1}}{x}+\ln\left(x+\sqrt{x^2+1}\right)$$

$$y'=\frac{-\cancel{\tfrac{1}{2}}(x^2+1)^{-\frac{1}{2}}(\cancel{2}x)(x)+\sqrt{x^2+1}}{x^2}+\frac{1+\cancel{\tfrac{1}{2}}(x^2+1)^{-\frac{1}{2}}(\cancel{2}x)}{x+\sqrt{x^2+1}}$$

$$=\frac{\dfrac{-x^2}{\sqrt{x^2+1}}+\sqrt{x^2+1}}{x^2}+\frac{1+\dfrac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}}$$

$$=\frac{\cancel{-x^2}+\cancel{x^2}+1}{x^2\sqrt{x^2+1}}+\frac{\cancel{\sqrt{x^2+1}+x}}{\cancel{\left(x+\sqrt{x^2+1}\right)}\sqrt{x^2+1}}$$

$$=\frac{1}{x^2\sqrt{x^2+1}}+\frac{x^2}{x^2\sqrt{x^2+1}}$$

$$=\frac{1+x^2}{x^2\sqrt{x^2+1}}=\frac{\cancel{(1+x^2)}\sqrt{x^2+1}}{x^2\cancel{(x^2+1)}}$$

$$=\frac{\sqrt{x^2+1}}{x^2}$$

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$$f(x) = \int_{2}^{\ln(2x)} (t+1)\,dt$$

$$f'(x) = (\ln 2x + 1) \cdot \frac{\cancel{2}}{\cancel{2}x} - 0$$

$$= \frac{\ln 2x + 1}{x}$$

73 $f(x) = \ln\sqrt{1+\sin^2 x}$ $\left(\frac{\pi}{4}, \ln\sqrt{\frac{3}{2}}\right)$

$f(x) = \frac{1}{2}\ln(1+\sin^2 x)$

$f'(x) = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\sin x\cos x}{1+\sin^2 x}$

$f'\left(\frac{\pi}{4}\right) = \frac{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}}{1+\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$

$y - \ln\sqrt{\frac{3}{2}} = \frac{1}{3}\left(x - \frac{\pi}{4}\right)$

79 $x+y-1=\ln(x^2+y^2)$ $(1,0)$

$$1+\frac{dy}{dx}=\frac{2x+2y\frac{dy}{dx}}{x^2+y^2}$$

$$1+\frac{dy}{dx}=\frac{2+0}{1+0}$$

$$\frac{dy}{dx}=1$$

$$y-0=1(x-1)$$

5.2a Integrals involving Natural Logs!

At the end of this lesson you will be able to:

> Antidifferentiate functions that involve natural logarithms
> Know when to use logs and when not to

2nd Warmup: Can you figure it out??

$$\int \frac{2x}{x^2+1}\,dx =$$

$$\int \frac{1}{u}\,du = \ln|u| + C$$

$$= \ln(x^2+1) + C$$

$u = x^2+1$

$du = 2x\,dx$

$$\frac{u'}{u}$$

Natural Log Rule! $$\int \frac{1}{x}\,dx = \ln|x| + C$$

ex) $$\int \frac{1}{x\ln x}\,dx = \int \frac{1}{u}\,du$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \ln|u| + C$$

$$= \ln|\ln x| + C$$

1) $\int \frac{1}{3x+2}\,dx = \int \frac{1}{3}\cdot\frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|3x+2| + C$

$u = 3x+2$

$du = 3\,dx$

$\frac{1}{3}du = dx$

2) $\int \frac{x^2}{3-x^3}\,dx = \int -\frac{1}{3}\frac{1}{u}\,du = -\frac{1}{3}\ln|u| + C$

$= -\frac{1}{3}\ln|3-x^3| + C$

$u = 3-x^3$

$du = -3x^2\,dx$

$-\frac{1}{3}du = x^2\,dx$

3) $\int \frac{x}{\sqrt{9-x^2}}\,dx = \int -\frac{1}{2}\cdot\frac{1}{\sqrt{u}}\,du = \int -\frac{1}{2}u^{-\frac{1}{2}}\,du$

$= -u^{\frac{1}{2}} + C = -\sqrt{9-x^2} + C$

$u = 9-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

4) $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{1}{u}\,du$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$= -\ln|u| + C$

$= -\ln|\cos x| + C$

But wait, there's more! Sometimes you encounter a problem like this. This is a treat because you get to brush off some of those old algebra skills!

Do this whenever the degree of top $\geq$ degree of bottom

Ex) $$\int \frac{x^2+3x+1}{x+1}\,dx = \int\left(x+2+\frac{-1}{x+1}\right)dx$$

$$= \frac{1}{2}x^2+2x-\ln|x+1|+C$$

Long division

$$\begin{array}{r} x+2 \\ x+1\enclose{longdiv}{x^2+3x+1} \\ -(x^2+x) \\ \hline 2x+1 \\ -(2x+2) \\ \hline -1 \end{array}$$

$$\begin{array}{c|ccc} -1 & 1 & 3 & 1 \\ & & -1 & -2 \\ \hline & 1 & 2 & -1 \end{array}$$

$$x+2+\frac{-1}{x+1}$$

Ex) $\int \frac{x^2-5x+7}{x+2}dx = \int\left(x-7+\frac{21}{x+2}\right)dx$

$$\begin{array}{r|rrr} -2 & 1 & -5 & 7 \\ & & -2 & 14 \\ \hline & 1 & -7 & 21 \end{array}$$

$= \frac{1}{2}x^2 - 7x + 21\ln|x+2| + C$

ex) $\int \frac{x^3-4x+1}{x-3}dx = \int\left(x^2+3x+5+\frac{16}{x-3}\right)dx$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -4 & 1 \\ & & 3 & 9 & 15 \\ \hline & 1 & 3 & 5 & 16 \end{array}$$

$= \frac{1}{3}x^3 + \frac{3}{2}x^2 + 5x + 16\ln|x-3| + C$

Some new formulas to memorize!!

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

Ex) $\int \sec(7x)\,dx = \int \frac{1}{7}\sec u\,du$

$$= \frac{1}{7}\ln|\sec u + \tan u| + C$$

$$= \frac{1}{7}\ln|\sec 7x + \tan 7x| + C$$

$u = 7x$

$du = 7\,dx$

$\frac{1}{7}du = dx$

What have we learned?

- Can I antidifferentiate a function in the form of 1/u ?
- Can I state the antiderivatives of all six major trig functions?