WARMUP!! (NO CALCULATORS!!)

A table of values is given below. Use this table to answer the following:

If y = lnx, then dy/dx = $\frac{1}{x}$

x	lnx	(lnx)'
1	0	1
2	0.6931	0.5
3	1.0986	0.33333
4	1.3863	0.25
5	1.6094	0.2

5.1b Derivatives of Natural Logs!

At the end of this lesson you will be able to:

> Differentiate functions involving natural logs
> Use logarithmic differentiation to simplify derivatives of complex functions

So, you've figured out that the derivative of $y = \ln x$ is $1/x$. Can you use this knowledge to differentiate $y = \ln(x^2 - 5)$?

$$\frac{dy}{dx} = \frac{2x}{x^2-5}$$

In general, if $y = \ln(u)$, then $dy/dx = \frac{u'}{u}$ $\frac{u'}{u}$

You try! Find dy/dx if y =

1) $\ln(x + 3)$ $\quad y' = \dfrac{1}{x+3}$

2) $\ln(3x)$ $\quad y' = \dfrac{3}{3x} = \dfrac{1}{x}$

3) $\ln(2x^3 + 4)$ $\quad y' = \dfrac{6x^2}{2x^3+4} = \dfrac{3x^2}{x^3+2}$

4) $\ln(x^2)$ $\quad y' = \dfrac{2x}{x^2} = \dfrac{2}{x}$

5) $\ln(x^3)$ $\quad y' = \dfrac{3x^2}{x^3} = \dfrac{3}{x}$

6) $\ln(\cos x)$ $\quad y' = \dfrac{-\sin x}{\cos x} = -\tan x$

7) $\ln(\sec x)$ $\quad y' = \dfrac{\sec x \tan x}{\sec x} = \tan x$

8) $\ln(\tan x)$ $\quad y' = \dfrac{\sec^2 x}{\tan x} = \dfrac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \sec x \csc x$

Don't forget, you can use properties of logs to break up a function before finding the derivative! Make your life easier and do this!

ex) $y = \ln\left(\frac{x^2}{\sqrt{2x^3}}\right)$, find y'

$$y = 2\ln x - \frac{1}{2}\ln(2x^3)$$

$$y' = \frac{2}{x} - \frac{1}{2}\cdot\frac{6x^2}{2x^3}$$

$$= \frac{2}{x} - \frac{3}{2x} = \frac{1}{2x}$$

You try!

ex) $y = \ln(x^2\sqrt{3x+5})$, find y'

$$y = 2\ln x + \frac{1}{2}\ln(3x+5)$$

$$y' = \frac{2}{x} + \frac{1}{2} \cdot \frac{3}{3x+5}$$

Logarithmic Differentiation!

This is the process of using logs to simplify non-logarithmic functions prior to differentiating.

ex) Find the derivative of $y = \frac{(x-2)^2}{\sqrt{x^2+1}}$

(See if you can figure this out in your groups. Hint: start by taking the natural log of both sides and then using properties to break up the log. Don't simplify.)

$$\ln y = \ln\left[\frac{(x-2)^2}{\sqrt{x^2+1}}\right]$$

$$\ln y = 2\ln(x-2) - \frac{1}{2}\ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x-2} - \frac{1}{2}\cdot\frac{2x}{x^2+1}$$

$$\frac{dy}{dx} = \left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]\cdot y$$

$$= \left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]\left[\frac{(x-2)^2}{\sqrt{x^2+1}}\right]$$

Time for some challenge! Use log differentiation to find y' if $y = x^{(2x+3)}$

$$\ln y = \ln x^{(2x+3)}$$

$$\ln y = (2x+3)\ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 2\ln x + (2x+3)\cdot\frac{1}{x}$$

$$\frac{dy}{dx} = \left[2\ln x + \frac{2x+3}{x}\right]\left[x^{(2x+3)}\right]$$

One last note!

$$\frac{d}{dx}\ln|u| = \frac{d}{dx}\ln u = \frac{u'}{u}$$

ex) If $y = \ln|x^2 - 5|$,

then $y' = \dfrac{2x}{x^2-5}$

What have we learned?

- What is the derivative of $y = \ln x$?
- What is the derivative of $y = \ln u$?
- Can I use properties of logs to differentiate non-log functions?