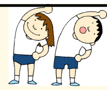
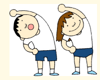
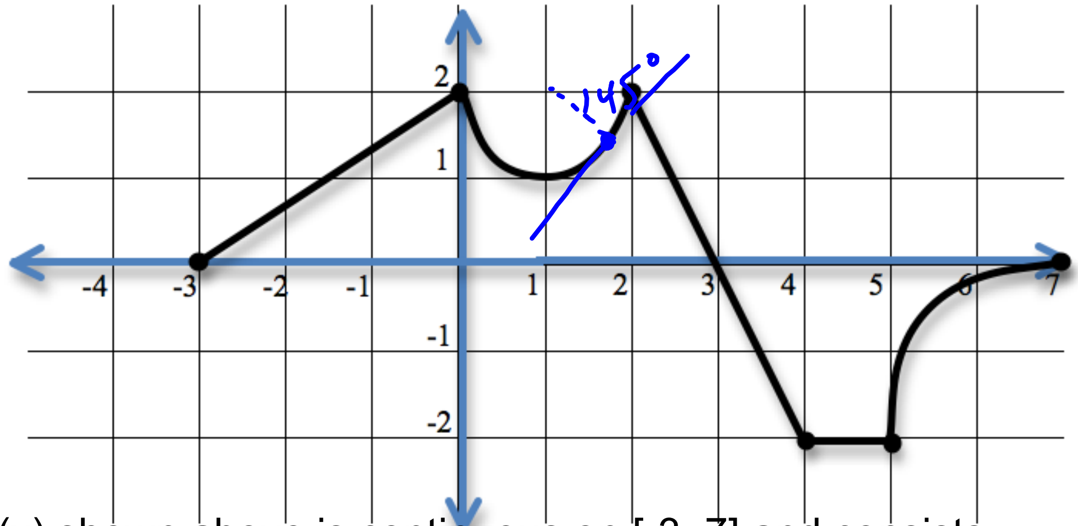


WARM UP!!





f(x) shown above is continuous on [-3, 7] and consists of 3 line segments, a semicircle and a quarter circle. Use this to find the following:

1) $f(-3)$

2) $f'(4)$

3) $f(1)$

4) $\int_4^5 f(x)dx$

5) $\int_{-1}^{-1} f(x)dx$

6) $f\,(0)$

7) $f'(3)$

8) $\int_0^2 f(x)dx$

9) $f(4)$

10) $f'(-1)$

11) $\int_{-3}^2 f(x)dx$

12) $\int_2^4 f(x)dx$

13) $f''(3)$

14) $\int_5^4 f(x)dx$

15) $f'(1)$

16) $f'(7)$

17) $f'\left(1 + \frac{\sqrt{2}}{2}\right)$

18) $\int_{-3}^7 f(x)dx$

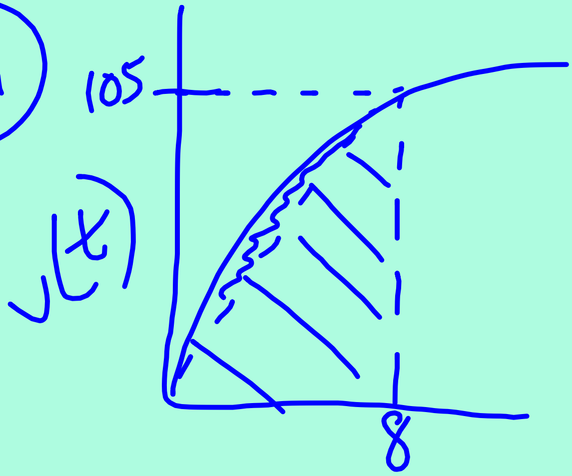
1) 0 2) und 3) 1 4) -2 5) 0

6) 2 7) -2 8) 4 - π/2 9) -2 10) 2/3

11) 7 - π/2 12) 0 13) 0 14) 2

15) 0 16) und 17) 1 18) 1 + π/2

51



$$\text{distance} = \int_0^8 |v(t)|\,dt$$

$$\approx \frac{(105)(8)}{2} = 420$$

(69) $F(x)=\int_1^x \frac{10}{v^2}\,dv$

$= \left.\frac{-10}{v}\right|_1^x$

$= \frac{-10}{x} - \frac{-10}{1} = \frac{-10}{x} + 10$

87 Find $F'(x)$ if

$$F(x) = \int_{x}^{x+2} (4t+1)\,dt$$

$$F'(x) = (4(x+2)+1)(1) - (4x+1)(1)$$

$$= 4x + 8 + 1 - 4x - 1$$

$$= 8$$

91 $F'(x)$ if

$$F(x) = \int_0^{x^3} \sin t^2 \, dt$$

$$F'(x) = (\sin x^6)(3x^2) - 0$$

$$= 3x^2 \sin x^6$$

4.5a U-Substitution!!

At the end of this lesson you will be able to:

> Determine when u-substitution can be used as a method to integrate functions

> Use u-substitution appropriately

2nd warmup: Evaluate $\int x(x^2+1)\,dx$

$$= \int (x^3 + x)\,dx$$

$$= \frac{1}{4}x^4 + \frac{1}{2}x^2 + C$$

Great! Now suppose you had to evaluate $\int \underline{x}(\underline{x^2+1})^{15}\,dx$! What would you do?

U-substitution is a method that allows us to integrate functions that were originally created by differentiating with chain rule. It is basically un-doing the chain rule.

To use this method, you need to start by looking carefully at the function and determining if you see a function and it's derivative somewhere in the problem.

(The coefficient of the derivative doesn't have to be correct.)

Once you find this, let u = the function portion and rewrite the integral in terms of u.

$$\int x(x^2+1)^{15}\,dx = \int \frac{1}{2}u^{15}\,du$$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\cdot\frac{1}{16}u^{16} + C$$

$$= \frac{1}{32}(x^2+1)^{16} + C$$

ex) $\int x^2 \sec^2(2x^3)\,dx = \int \frac{1}{6}\sec^2 u\,du$

$u = 2x^3$

$du = 6x^2\,dx$

$\frac{1}{6}du = x^2\,dx$

$= \frac{1}{6}\tan u + C$

$= \frac{1}{6}\tan(2x^3) + C$

ex) $\int 7x^3\sqrt{5-x^4}\,dx = \int -\frac{7}{4}u^{\frac{1}{2}}\,du$

$u = 5 - x^4$

$du = -4x^3\,dx$

$-\frac{1}{4}du = x^3\,dx$

$-\frac{7}{4}du = 7x^3\,dx$

$= -\frac{7}{4}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$

$= -\frac{7}{6}(5-x^4)^{\frac{3}{2}} + C$

Now U try!! For each of the following, rewrite each integral as a function of u.

(Do not integrate.)

1) $\int \sqrt{7x+2}\,dx$ $u=7x+2$ $\int \frac{1}{7} u^{\frac{1}{2}} du$

2) $\int \tan^5 x \sec^2 x\,dx$ $\int u^5 du$

$u=\tan x$

3) $\int \frac{x}{(x^2+8)^3}dx$ $\int \frac{1}{2} u^{-3} du$

$u=x^2+8$

4) $\int (2x+1)(x^2+x)dx$ $\int u\,du$

$u=x^2+x$

$du=(2x+1)dx$

5) $\int \sin^3 x \cos x\,dx$

$u=\sin x$ $\int u^3 du$

Remember, not every function requires u-substitution to integrate!

ex) $\int \frac{t^2 + 2t}{\sqrt{t}} dt = \int \left(t^{\frac{3}{2}} + 2t^{\frac{1}{2}}\right) dt$

$= \frac{2}{5}t^{\frac{5}{2}} + \frac{4}{3}t^{\frac{3}{2}} + C$

WHAT HAVE WE LEARNED?

- When is u-substitution necessary and appropriate?
- What do I need to look for when approaching an integral with a product or quotient?
- Can I rewrite an integral in terms of u?