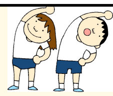
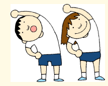


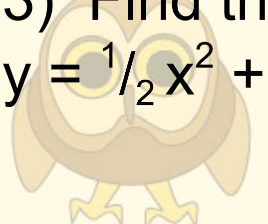
WARM UP!!



1) Write the sum as an integral $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{9}{n}\right)\left(-4+\frac{9i}{n}\right)^{3}$

2) Evaluate $\displaystyle\int_{3}^{7}(2x^2-3)\,dx$

3) Find the area under the curve
$y = {}^{1}/_{2}\,x^2 + 3x + 4$ from $x = -1$ to $x = 3$



4) The acceleration of an object moving horizontally is 3 in/sec^2. If $v(3) = 13$, find the total distance traveled from $t = 0$ to $t = 20$.

1) $\displaystyle\int_{-4}^{5} x^3\,dx$ 2) 596/3 3) 98/3 4) 680 in

4.4b Average Value and SFTC!

At the end of this lesson you will be able to:

- find the average value of a function over a given interval
- State what the SFTC is and apply it to simple problems

Average Value Theorem/Formula:

$$Average\ value = f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

What does this mean? How does it work?

Rewrite the formula as:

$$(b-a)\cdot f(c) = \int_a^b f(x)\,dx$$

(This rewrite is known as the Mean Value Theorem for Integrals.)

Can I see this visually? Sure!!

ex) Find the average value of $f(x) = x^2 + 1$ on [2, 5]. Then find 'c' guaranteed by the MVT for integrals.

$$\text{Avg Value} = f(c) = \frac{1}{5-2}\int_2^5 (x^2+1)\,dx$$

$$= \frac{1}{3}\left[\frac{1}{3}x^3 + x \Big|_2^5\right]$$

$$= \frac{1}{3}\left[\frac{125}{3} + 5 - \left(\frac{8}{3} + 2\right)\right]$$

$$= \frac{1}{3}\left[\frac{117}{3} + 3\right]$$

$$= \frac{39}{3} + 1 = 14 \leftarrow f(c)$$

To find c:

$$x^2 + 1 = 14 \qquad x = \pm\sqrt{13}$$

$$x^2 = 13 \qquad \boxed{c = \sqrt{13}}$$

You try! Find the average value of f(x) = cosx on [-π/3, π/3]. Then find 'c' guaranteed by the MVT for integrals. (No calc for the avg value, but you will probably need a calculator to find c.)

$$f(c) = \frac{1}{\frac{\pi}{3} - -\frac{\pi}{3}} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos x \, dx$$

$$= \frac{1}{\frac{2\pi}{3}} \left[\sin x \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \right]$$

$$= \frac{3}{2\pi}\left(\sin\frac{\pi}{3} - \sin -\frac{\pi}{3}\right) = \frac{3}{2\pi}\left(\frac{\sqrt{3}}{2} - -\frac{\sqrt{3}}{2}\right)$$

$$= \frac{3}{2\pi}\left(\sqrt{3}\right) = \frac{3\sqrt{3}}{2\pi}$$

find c:

$$\cos x = \frac{3\sqrt{3}}{2\pi}$$

$$x \approx .597, -.597$$
$$c \approx$$

The integral as a function!!

Suppose we kept the argument of the integral static, but changed the limits of the integral to be variables. This would create an area finding function for that function. (Refer to geogebra example from 3 screens back.)

(This is especially useful for applications. If the function represents something in terms of time and we want to find the integral with a variety of time inputs. You will see many of these later on.)

ex) Find F(-π), F(0), and F(π) if

$$F(x) = \int_0^x (\sin t)dt$$

$$F(-\pi) = \int_0^{-\pi} \sin t\, dt$$

$$= -\int_{-\pi}^{0} \sin t\, dt = --\cos t \Big|_{-\pi}^{0}$$

$$= \cos 0 - \cos(-\pi)$$

$$= 1 - -1 = 2$$

$$F(0) = \int_0^0 \sin t\, dt = 0$$

$$F(\pi) = \int_0^{\pi} \sin t\, dt = -\cos t \Big|_0^{\pi}$$

$$= -\cos \pi - -\cos 0$$

$$= 1 + 1 = 2$$

Second Fundamental Theorem of Calculus

(SFTC)

$$\frac{d}{dx}\left[\int_{q(x)}^{p(x)} f(t)\,dt\right] = f(p(x))\cdot p'(x) - f(q(x))\cdot q'(x)$$

ex) Evaluate F(x). Then find F'(x).

$$F(x) = \int_{x}^{x^2} (2t+3)\,dt = t^2+3t\Big|_{x}^{x^2}$$

$$= x^4+3x^2-(x^2+3x) = x^4+2x^2-3x$$

$$F'(x) = 4x^3+4x-3$$

Let's do it again using the SFTC!!

$$F'(x) = (2x^2+3)2x - (2x+3)(1)$$

$$= 4x^3+6x-2x-3$$

$$= 4x^3+4x-3$$

Evaluate each of the following:

1) $\frac{d}{dx}\left[\int_{-3}^{x}\sqrt{t^2+4}\,dt\right] = \sqrt{x^2+4}$

2) $\frac{d}{dx}\left[\int_{3}^{x^2}\sqrt{t-1}\,dt\right] = 2x\sqrt{x^2-1}$

3) $\frac{d}{dx}\left[\int_{10}^{x^2}\sqrt{t-1}\,dt\right] = 2x\sqrt{x^2-1}$

4) $\frac{d}{dx}\left[\int_{3x}^{0}\frac{1}{t+2}\,dt\right] = -\frac{3}{3x+2}$

What have we learned??

- How do I find the average value of a function?
- What does the average value of a function mean?
- When does the SFTC apply?
- How do I apply the SFTC?