Get those brain cells movin' by pondering this one...

$$\int x\sqrt{x+1}\,dx$$

Today's Essential Learning Targets:

I can solve a definite integral using u-substitution

I can solve both definite and indefinite integrals using the "u-substitution change of variable" method

Let's do a quick review from yesterday...

1) $\int csc^2\left(\frac{x}{2}\right)dx = \int 2csc^2(u)\,du = -2\cot u + C$

$= -2\cot\left(\frac{x}{2}\right) + C$

$u = \frac{1}{2}x$

$du = \frac{1}{2}dx$

$2du = dx$

2) $\int x\sqrt{1-x^2}\,dx = \int -\frac{1}{2}u^{\frac{1}{2}}\,du$

$u = 1 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= -\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$

$= -\frac{1}{3}\sqrt{(1-x^2)^3} + C$

Today we extend u-substitution to definite integrals. There are two ways to approach these questions:

Change the interval	Don't Change the Interval

Change the interval

$u = 1 + x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int_{x=0}^{x=3} x\sqrt{1+x^2}\,dx = \int_{u=1}^{u=10} \frac{1}{2}u^{\frac{1}{2}}\,du$$

$$= \frac{1}{3}u^{\frac{3}{2}}\Big|_1^{10} = \frac{1}{3}\left(10^{\frac{3}{2}} - 1\right)$$

Don't Change the Interval

$$\int_0^3 x\sqrt{1+x^2}\,dx$$

$u = 1 + x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int_{x=0}^{x=3} \frac{1}{2}u^{\frac{1}{2}}\,du = \frac{1}{3}u^{\frac{3}{2}}\Big|_{x=0}^{x=3}$$

$$= \frac{1}{3}\left(1+x^2\right)^{\frac{3}{2}}\Big|_0^3$$

$$= \frac{1}{3}\left(10^{\frac{3}{2}} - 1\right)$$

You try...

$$\int_{-1}^{2} \frac{4x}{\sqrt{1+3x^2}}\,dx = \int_{u=4}^{u=13} \frac{2}{3} u^{-\frac{1}{2}}\,du = \frac{4}{3} u^{\frac{1}{2}} \Big|_{4}^{13}$$

$$= \frac{4}{3}\left(\sqrt{13} - 2\right)$$

$u = 1 + 3x^2$

$du = 6x\,dx$

$\frac{2}{3}du = 4x\,dx$

Let's go back to the one we started the hour with...

$$\int x\sqrt{x+1}\,dx = \int (u-1)u^{\frac{1}{2}}\,du = \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$$

$u = x+1$

$du = dx$

$x = u-1$

$$= \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + C$$

$$= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C$$

How about this one...

$$\int_{-2}^{6} x^2\sqrt[3]{x+2}\,dx = \int_{u=0}^{u=8} (u^2-4u+4)\left(u^{\frac{1}{3}}\right) du$$

$u = x+2$

$du = dx$

$x = u-2$

$x^2 = (u-2)^2 = u^2-4u+4$

$$= \int_0^8 \left(u^{\frac{7}{3}} - 4u^{\frac{4}{3}} + 4u^{\frac{1}{3}}\right) du$$

$$= \left(\frac{3}{10}u^{\frac{10}{3}} - \frac{12}{7}u^{\frac{7}{3}} + 3u^{\frac{4}{3}}\right)\Big|_0^8$$

$$= \frac{3}{10}\left(2^{10}\right) - \frac{12}{7}\left(2^{7}\right) + 3\left(2^{4}\right) - 0$$

$$= \frac{3072}{10} - \frac{1536}{7} + 48$$

$$= \frac{4752}{35} \approx 135.771$$

And this one...

$$\int x\sqrt{2x-1}\,dx$$

This is from the 2012 AP AB Exam

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

What have we learned?

How to evaluate a definite integral using u-substitution

How to evaluate a definite and indefinite integral using the "u-substitution change of variable" method