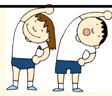
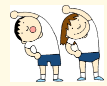
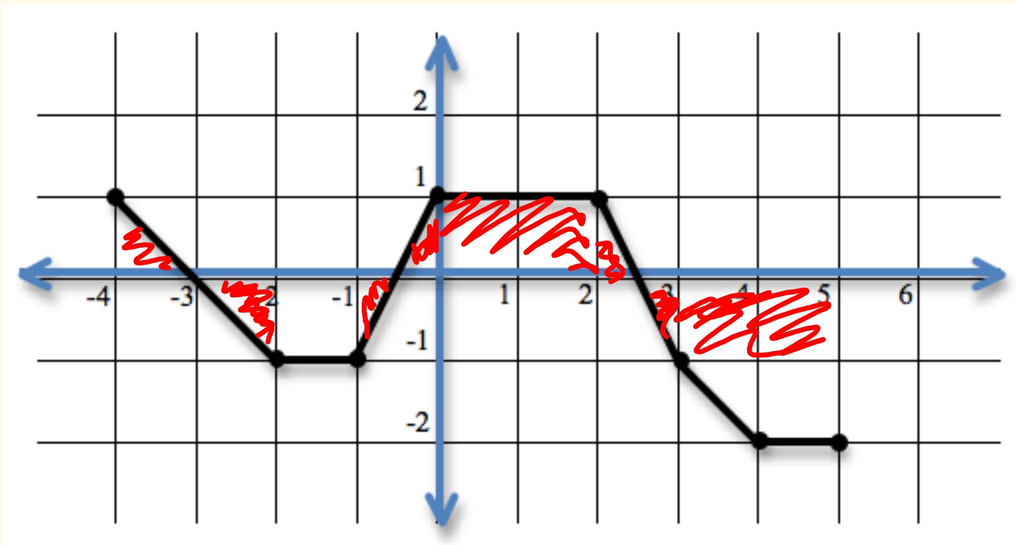


WARM UP!!





1) $f'(1) = 0$

4) $\int_{0}^{2} f(x)dx = 2$

7) $\int_{4}^{5} f(x)dx = -2$

2) $f'(5) =$ und

5) $\int_{-4}^{-3} f(x)dx = \frac{1}{2}$

8) $\int_{-3}^{-2} f(x)dx = -\frac{1}{2}$

3) $f'(-3) = -1$

6) $\int_{-1}^{0} f(x)dx = 0$

9) $\int_{-4}^{5} f(x)dx = -\frac{5}{2}$

4.4a Definite Integrals!

Essential Learning Targets

- Evaluate definite integrals of functions containing removable or jump discontinuities
- Uses the Fundamental Theorem of Calculus to define new functions
- Uses the Fundamental Theorem of Calculus to evaluate definite integrals
- Solves problems in which the graphical, numerical, analytical, and verbal representations of a function provide information about a new function found using the Second Fundamental Theorem of Calculus

2nd warmup: Suppose my velocity function is $v(t) = 3t^2 + 5$ for $t > 0$. How far did I travel from $t = 1$ to $t = 3$?

$$s(t) = \int v(t)\,dt = t^3 + 5t + C$$

$$s(1) = 6 + C$$

$$s(3) = 42 + C$$

$$s(3) - s(1) = 42 + C - (6 + C) = 36$$

$$\int_1^3 (3t^2 + 5)\,dt = t^3 + 5t \Big|_1^3$$

$$= 27 + 15 - (1 + 5) = \underline{36}$$

$$\int_a^b v(t)\,dt = p(t)\Big|_a^b = p(b) - p(a)$$

$$\int_a^b W'(t)\,dt = W(t)\Big|_a^b = W(b) - W(a)$$

$$\underbrace{\int (3t^2 + 5)\,dt}_{\text{indefinite integral}} = \underbrace{t^3 + 5t + C}_{\text{antiderivative}}$$

Fundamental Theorem of Calculus (FTC)

$$\int_a^b f(x)dx = F(b) - F(a)$$

(where F is the antiderivative of f)

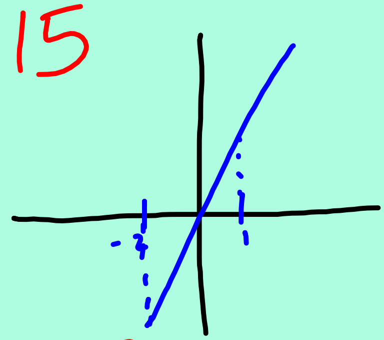
ex) Evaluate $\int_1^4 (3x^2 + 4x - 1)dx$

$$= x^3 + 2x^2 - x \Big|_1^4$$

$$= 64 + 32 - 4 - (1 + 2 - 1) = 90$$

While the indefinite integrals gives you an area finding function, the result of a definite integral is the actual area between 2 values of x (provided that the function is above the x-axis).

ex) $\int_1^4 2x\,dx =$ $x^2\Big|_1^4 = 16-1=15$



ex) $\int_{-2}^{1} 2x\,dx =$ $x^2\Big|_{-2}^{1} = 1-4=-3$

You try! Evaluate the following integrals using the FTC. If the integral is presented in sigma form, rewrite using integral notation and evaluate using the FTC.

1) $\int_{-2}^{5}(x^2+2)\,dx$

2) $\int_{0}^{\frac{\pi}{2}} \sin x\ dx$

3) $\int_{0}^{3} x^4 dx$

4) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{5}{n}\right)\left(-3+\frac{5i}{n}\right)^2 = \int_{-3}^{2} x^2\,dx = \frac{1}{3}x^3\Big|_{-3}^{2}$

$= \frac{8}{3} - -\frac{27}{3} = \frac{35}{3}$

5) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2}{n}\right)\sqrt{\frac{2i}{n}} = \int_{0}^{2}\sqrt{x}\,dx$

6) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{7}{n}\right)\left[4\left(\frac{7i}{n}-3\right)^3+5\left(\frac{7i}{n}-3\right)\right] = \int_{-3}^{4}(4x^3+5x)\,dx$

Answers

1) 175/3

2) 1

3) 243/5

4) 35/3

5) $\frac{4\sqrt{2}}{3}$

6) 385/2

Just for fun! Try this one!

$$|1-x^2| = \begin{cases} -(1-x^2), & x \geq 1 \\ 1-x^2, & -1 < x < 1 \\ -(1-x^2), & x \leq -1 \end{cases}$$

$$\int_{-3}^{2} |1-x^2|\,dx = \int_{-3}^{-1} -(1-x^2)\,dx + \int_{-1}^{1} (1-x^2)\,dx + \int_{1}^{2} (-1+x^2)\,dx$$

$$= -x+\tfrac{1}{3}x^3 \Big|_{-3}^{-1} + x-\tfrac{1}{3}x^3 \Big|_{-1}^{1} + -x+\tfrac{1}{3}x^3 \Big|_{1}^{2}$$

$$= 1-\tfrac{1}{3}-(3-9) + 1-\tfrac{1}{3}-(-1+\tfrac{1}{3}) + -2+\tfrac{8}{3}-(-1+\tfrac{1}{3})$$

$$= 1-\tfrac{1}{3}+6+1-\tfrac{1}{3}+1-\tfrac{1}{3}-2+\tfrac{8}{3}+1-\tfrac{1}{3}$$

$$= 8+\tfrac{4}{3} = \tfrac{28}{3}$$

Use the FTC to find values on the antiderivative at specific values of x.

$$\int_a^b f(x)dx = F(b) - F(a)$$

$$F(b) = F(a) + \int_a^b f(x)dx$$

new amount = starting amount + net change

ex) Suppose we know that $\int_{-1}^{3} f'(x)dx = 10$

If f(3) = 7, find f(-1).

$$\int_{-1}^{3} f'(x)dx = f(3) - f(-1)$$

$$10 = 7 - f(-1)$$

$$f(-1) = 7 - 10 = -3$$

ex) Suppose the area under a velocity curve from t = 0 to t = 6 is 25, and the object is always moving to the right. If the initial position is -3, find the position of the object at t = 6.

$$p(6) = p(0) + \int_0^6 v(t)\,dt$$

$$= -3 + 25$$

$$= \boxed{22}$$

What have we learned??

- What is the FTC?
- Can I use the FTC to solve equations involving integrals?