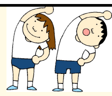
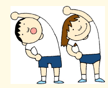
November 04, 2016

 # WARM UP!! 

1) $\int \frac{1}{x^4}\,dx = -\frac{1}{3x^3} + C$

2) $\int 7\,dt = 7t + C$

3) $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx = \frac{2}{3}\sqrt{x^3} + \sqrt{x} + C$

4) $\int \left(\sqrt{6x}\right) dx = \int \sqrt{6}\, x^{\frac{1}{2}}\,dx = \frac{2\sqrt{6}}{3}\sqrt{x^3} + C$

5) $\int (\theta^2 + \sec^2\theta)\,d\theta = \frac{1}{3}\theta^3 + \tan\theta + C$

6) $\int \frac{\sin x}{1 - \sin^2 x}\,dx = \int \frac{\sin x}{\cos^2 x}\,dx = \int \sec x \tan x\,dx = \sec x + C$

4.3a Area and Integrals!

Essential Learning Targets:

- Knows and applies the definition of a definite integral, and relates this to the limit of the Riemann sum
- Approximates definite integrals for functions represented graphically, numerically, algebraically and verbally
- Knows the connection between the definite integral and area and evaluates definite integrals using geometry when applicable
- Applies the properties of definite integrals including integrals of a constant times a function, the sum of two functions, reversal of limits and the integral of a function over adjacent intervals

Area under a curve

$$\lim_{n\to\infty}\sum_{i=1}^{n}(width)*f(left\ endpoint + width*i)$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(\frac{b-a}{n}\right)*f\left(a+\left(\frac{b-a}{n}\right)i\right)\right]$$

$$\int_a^b \underbrace{f(x)}_{\text{height of rectangle}}\underbrace{dx}_{\text{width of rectangle}}$$

This is called the <u>definite integral</u> of f(x) from a to b, where a is the 'lower limit' and b is the 'upper limit'.

ex) Evaluate $\int_{-2}^{3}(x+3)\,dx$ using limits.

Then check your answer by sketching a graph and using geometry.

$[a,b]=[-2,3] \quad f(x)=x+3$

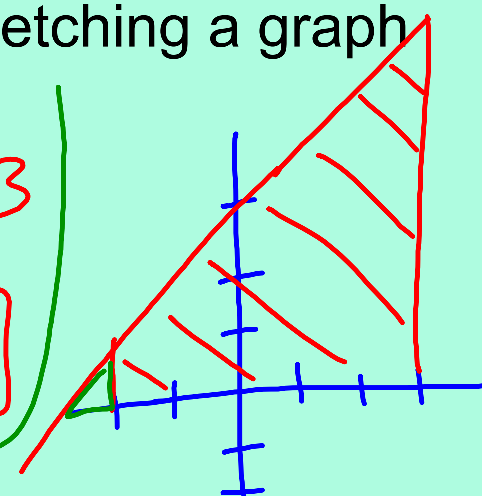
$$A=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{5}{n}\right)\left[\left(-2+\frac{5}{n}i\right)+3\right]$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{5}{n}\right)\left[\frac{5}{n}i+1\right]$$

$$=\lim_{n\to\infty}\left(\frac{5}{n}\right)\left[\frac{5}{n}\sum_{i=1}^{n}i+\sum_{i=1}^{n}1\right]$$

$$=\lim_{n\to\infty}\left(\frac{5}{n}\right)\left[\frac{5}{n}\cdot\frac{n(n+1)}{2}+n\right]$$

$$=\frac{25}{2}+5=\frac{35}{2}$$



$\frac{1}{2}(6)(6)-\frac{1}{2}(1)(1)$

$=18-\frac{1}{2}=17\frac{1}{2}=\frac{35}{2}$

Write the following limits as definite integrals.

$$\frac{b-a}{n} = \frac{5}{n}, \quad a=2, \quad b=7$$

ex. $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{5}{n}\left(2+\frac{5i}{n}\right)^3$

$$\int_2^7 x^3\,dx = \int_0^5 (2+x)^3\,dx = \int_1^6 (1+x)^3\,dx = \int_n^{n+5} (2-n+x)^3\,dx$$

per Grace Lofstrom

you try! $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{2}{n}\sqrt[3]{16-\frac{4i^2}{n^2}}$

$$\frac{b-a}{n} = \frac{2}{n}$$

$$y = \sqrt[3]{16-x^2}$$

$$\int_0^2 \sqrt[3]{16-x^2}\,dx$$

A few integral properties:

$$\int_a^a f(x)dx = 0$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \quad a \le c \le b$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\int_a^b c\,f(x)dx = c\int_a^b f(x)dx$$

ex) Suppose $\int_2^6 (x^3)dx = 320$, $\int_2^6 x\,dx = 16$, $\int_2^6 dx = 4$

Use this information to evaluate $\int_2^6 \left(\frac{1}{2}x^3 - 3x + 2\right) dx$

$$\int_2^6 \left(\frac{1}{2}x^3 - 3x + 2\right) dx = \frac{1}{2}\int_2^6 x^3 dx - 3\int_2^6 x\,dx + 2\int_2^6 dx$$

$$= \frac{1}{2}(320) - 3(16) + 2(4)$$

$$= 160 - 48 + 8$$

$$= 120$$

ex) Suppose $\int_0^2 x^3 dx = 4$ and $\int_2^6 x^3 dx = 320$

Use this information to evaluate

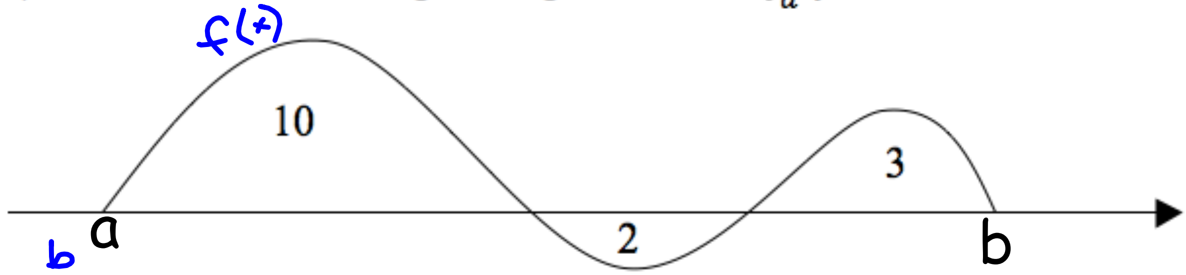
1) $\int_0^6 -2x^3 dx = -2\int_0^6 x^3 dx = -2\left[\int_0^2 x^3 dx + \int_2^6 x^3 dx\right] = -2(324)$
$= -648$

2) $\int_2^0 x^3 dx = -\int_0^2 x^3 dx = -4$

3) $\int_2^2 x^3 dx = 0$

An integral yields a positive answer for areas above the x-axis and a negative answer for areas below the x-axis.

Ex) The area for each region is given. Find $\int_a^b f(x)dx$



$$\int_a^b f(x)\,dx = 10 - 2 + 3 = 11$$

What have we learned??

- What does the ∫ symbol represent?
- What is the outcome of a definite integral?
- Can I break an integral into a sum of multiple integrals? Why?

0	8	16	25	34	44	. . .
35	43	48	50	48	47	

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