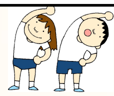
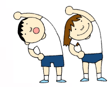


WARM UP!!



Find particular solutions to each of the following differential equations:

1) $f'(x) = 6x - 8x^3$, $f(2) = 3$

2) $f''(x) = x^2$, $f'(0) = 6$, $f(0) = 0$

3) $f''(x) = \sin x$, $f'(0) = 1$, $f(0) = 6$

1) $f(x) = 3x^2 - 2x^4 + 23$

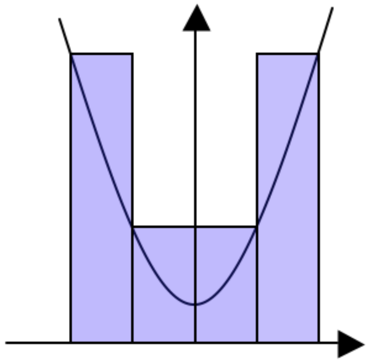
2) $f(x) = 1/12\, x^4 + 6x$

3) $f(x) = -\sin x + 2x + 6$

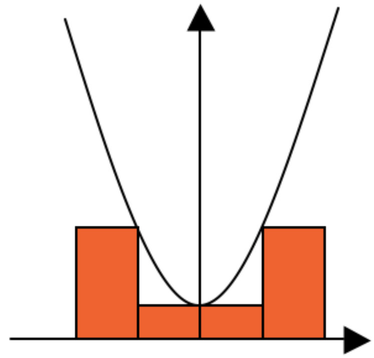
4.2a Sigma Notation and Area Approximation!

Essential Learning Target

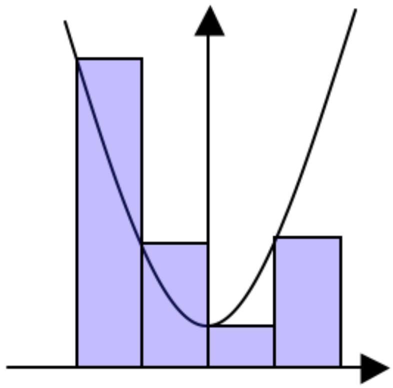
Compute left, right and midpoint Riemann sums using either uniform or non-uniform partitions



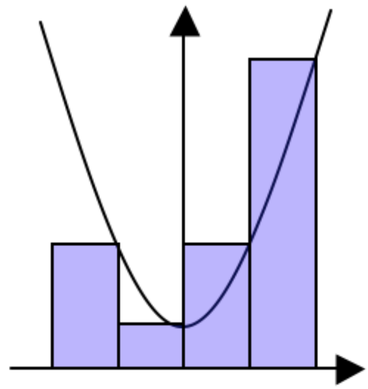
Upper or circumscribed rectangles will always have an area greater than that under the curve



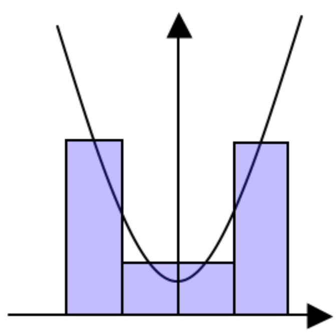
Lower or inscribed rect's will always have an area less than that under the curve



Left-handed rect's will always have a height equal to the value of the function on the left side

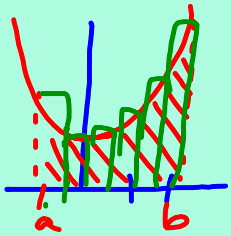


Right-handed rect's will always have a height equal to the value of the function on the right side



Midpoint rect's will always have a height equal to the value of the function at the midpoint of the rectangle

Ex) Use 6 rectangles to estimate upper, lower, left-handed and right-handed sums for the area bounded by $x = -1$, $y = 0$, $x = 2$, and $y = x^2 + 1$. Then use 3 rectangles to estimate the midpoint approximation of the area.



Determine the width of each rectangle:

$$\text{width} = \frac{b - a}{n}$$ (for rectangles of equal width only)

n → # of rectangles

left endpoint (a) = -1

right endpoint (b) = 2

Make a table!

x	-1	-1/2	0	1/2	1	3/2	2
f(x)	2	5/4	1	5/4	2	13/4	5

Upper: $\frac{1}{2}\left[2 + \frac{5}{4} + \frac{5}{4} + 2 + \frac{13}{4} + 5\right] = \frac{59}{8}$ or 7.375 (width)

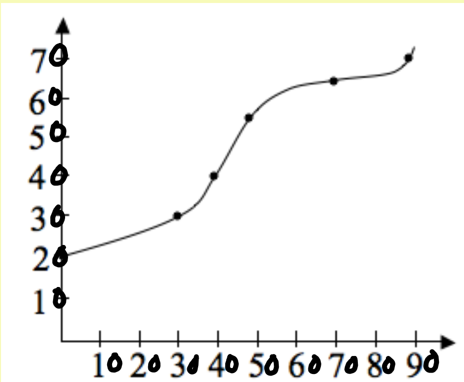
Lower: $\frac{1}{2}\left[\frac{5}{4} + 1 + 1 + \frac{5}{4} + 2 + \frac{13}{4}\right] = \frac{39}{8}$ or 4.875

Left: $\frac{1}{2}\left[2 + \frac{5}{4} + 1 + \frac{5}{4} + 2 + \frac{13}{4}\right] = \frac{43}{8}$ or 5.375

Right: $\frac{1}{2}\left[\frac{5}{4} + 1 + \frac{5}{4} + 2 + \frac{13}{4} + 5\right] = \frac{55}{8}$ or 6.875

Midpoint: (3 of them) $1\left[\frac{5}{4} + \frac{5}{4} + \frac{13}{4}\right] = \frac{23}{4} = 5.75$

2003 #3 (calculator question)



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

c) Approximate the value of the area under the curve using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of the actual area? Explain your reasoning.

Area $\approx 30(20)+10(30)+10(40)+20(55)+20(65)$

$= 3700$ gallons

less b/c left-handed rect's on an increasing function

Sigma notation!!

Upper bound

$$\sum_{i=1}^{4} a_i = a_1 + a_2 + a_3 + a_4$$

index lower bound

Ex) $\sum_{i=2}^{4} i^2 =$ $4 + 9 + 16 = 29$

Write the following in sigma notation:

1. $$\left[\left(\frac{8}{x+3}\right)+\left(\frac{8}{x+4}\right)+\ldots+\left(\frac{8}{x+9}\right)\right]$$

$$\sum_{i=3}^{9} \frac{8}{x+i}$$

2. $$\left[\left(\frac{4}{n}\right)\left(5+\frac{4}{n}\right)^{3}+\ldots+\left(\frac{4}{n}\right)\left(5+\frac{4n}{n}\right)^{3}\right]$$

$$\sum_{i=1}^{n} \left(\frac{4}{n}\right)\left(5+\frac{4i}{n}\right)^{3}$$

Formulas to know (memorize) !!

1) $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$

2) $\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

3) $\sum_{i=1}^{n} c = cn$

4) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

5) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

6) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Side note :)

Carl Friedrich Gauss

1777 - 1855 (German)

Discovered the sum of n integers (formula 4) when he was 8 years old. He was very conceited (although somewhat rightfully so) and refused to let his sons become mathematicians because he knew they wouldn't be as good as him and he didn't want them to diminish his name. He was one of the first to explore the possibility of non-euclidian geometry, but never published his findings. You might see his name come up when you study Gaussian elimination with matrices.

Ex) $\sum_{i=1}^{40}(i+2)^2 = \sum_{i=1}^{40}(i^2+4i+4)$

$= \sum_{i=1}^{40} i^2 + \sum_{i=1}^{40} 4i + \sum_{i=1}^{40} 4$

$= \sum_{i=1}^{40} i^2 + 4\sum_{i=1}^{40} i + \sum_{i=1}^{40} 4$

$= \frac{(40)(41)(81)}{6} + 4\cdot\frac{(40)(41)}{2} + 4(40)$

$= 22140 + 3280 + 160$

$= \boxed{25580}$

You try!

Ex) $\sum_{i=1}^{80}(3i^2+2)=$

$$3\sum_{i=1}^{80} i^2 + \sum_{i=1}^{80} 2$$

$$= \frac{3(80)(81)(161)}{6} + 2(80)$$

$$= \boxed{521800}$$

Limit review!

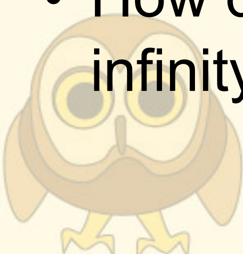
Can you figure out the value of these limits without doing any work at all?

$$\lim_{n\to\infty} \frac{4}{n^2} \cdot \frac{n(n+1)}{2} =$$

$$\lim_{n\to\infty} \frac{64}{n^3} \left(\frac{n(n)(n+1)}{2} - \frac{n(n+1)(2n+1)}{6} \right) =$$

What have we learned??

- How do I compute left, right-handed and midpoint approximations for area under a curve?
- What are the summation formulas for i, i^2, and i^3?
- How do I find a limit as x approaches infinity?



November 02, 2016