A.P. Calculus AB **Quiz 4-1 Review** **No Calculators**

Find the most general antiderivative of $h(x)$.

1. $h(x) = 5x^4 - 5x^2 + \pi + \frac{1}{2\sqrt{x}} + \frac{1}{3x^3}$ $= 5x^4 - 5x^2 + \pi + \frac{x^{-1/2}}{2} + \frac{1}{3}x^{-3}$

$$\int h(x)\,dx = \frac{5x^5}{5} - \frac{5x^3}{3} + \pi x + \frac{1}{2}\left(\frac{x^{1/2}}{1/2}\right) + \frac{1}{3}\left(\frac{x^{-2}}{-2}\right) + C = \boxed{x^5 - \frac{5x^3}{3} + \pi x + \sqrt{x} - \frac{1}{6x^2} + C}$$

2. $h(x) = \frac{-8}{\sqrt[3]{x}} - 2\csc^2 x$ $= -8x^{-1/3} - 2\csc^2 x$

$$\int h(x)\,dx = -8\left(\frac{x^{2/3}}{2/3}\right) - 2(-\cot x)$$

$$= -8\left(\tfrac{3}{2}\right)\left(x^{2/3}\right) + 2\cot x + C$$

$$= \boxed{-12x^{2/3} + 2\cot x + C}$$

3. $h(x) = \frac{7}{\sqrt[5]{x}} - \frac{\sqrt[4]{x}}{13} + 10$ $= 7x^{-1/5} - \frac{1}{13}x^{1/4} + 10$

$$\int h(x)\,dx = \frac{7x^{4/5}}{4/5} - \frac{1}{13}\left(\frac{x^{5/4}}{5/4}\right) + 10x + C =$$

$$7\left(\tfrac{5}{4}\right)x^{4/5} - \tfrac{1}{13}\left(\tfrac{4}{5}\right)x^{5/4} + 10x + C$$

$$\boxed{\frac{35}{4}x^{4/5} - \frac{4}{65}x^{5/4} + 10x + C}$$

4. $h(x) = \frac{2x - 6x^3 + x^2}{\sqrt[3]{x}}$ $= \frac{2x}{x^{1/3}} - \frac{6x^3}{x^{1/3}} + \frac{x^2}{x^{1/3}} = 2x^{2/3} - 6x^{8/3} + x^{5/3}$

$$\int h(x)\,dx = 2\left(\frac{x^{5/3}}{5/3}\right) - 6\left(\frac{x^{11/3}}{11/3}\right) + \frac{x^{8/3}}{8/3} + C$$

$$= 2\left(\tfrac{3}{5}\right)x^{5/3} - 6\left(\tfrac{3}{11}\right)x^{11/3} + \tfrac{3}{8}x^{8/3} + C$$

$$\boxed{\frac{6x^{5/3}}{5} - \frac{18x^{11/3}}{11} + \frac{3x^{8/3}}{8} + C}$$

5. h(x) = -2cosx + 5sinx – 5*cscxcotx*

$$\int h(x)\,dx = -2(\sin x) + 5(-\cos x) - 5(-\csc x) + C$$

$$= \boxed{-2\sin x - 5\cos x + 5\csc x + C}$$

6. Find the most general expression of $f(x)$ if $f''(x) = 9x^2 - 5x + 2$.

$$f'(x) = \frac{9x^3}{3} - \frac{5x^2}{2} + 2x + c = 3x^3 - \frac{5x^2}{2} + 2x + c$$

$$f(x) = \frac{3x^4}{4} - \frac{5}{2}\left(\frac{x^3}{3}\right) + \frac{2x^2}{2} + cx + k$$

$$= \boxed{\frac{3}{4}x^4 - \frac{5x^3}{6} + x^2 + cx + k}$$

7. Find the most general expression of $f(x)$ if $f''(x) = 4x^3 - 5x^2 + 3x - 6$.

$$f'(x) = \frac{4x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} - 6x + c$$

$$= x^4 - \frac{5x^3}{3} + \frac{3x^2}{2} - 6x + c$$

$$f(x) = \frac{x^5}{5} - \frac{5}{3}\left(\frac{x^4}{4}\right) + \frac{3}{2}\left(\frac{x^3}{3}\right) - \frac{6x^2}{2} + cx + k$$

$$\boxed{f(x) = \frac{x^5}{5} - \frac{5x^4}{12} + \frac{x^3}{2} - 3x^2 + cx + k}$$

8. Find the specific expression of $f(x)$ if $f(x) = \int g(x)dx$, $g(x) = 3x^2 - 4x$, and $f(-1) = 2$

$$\int g(x)dx = \int 3x^2 - 4x\,dx = \frac{3x^3}{3} - \frac{4x^2}{2} + c \rightarrow f(x) = x^3 - 2x^2 + c$$

$$f(-1) = (-1)^3 - 2(-1)^2 + c$$

$$2 = -1 - 2 + c$$

$$2 + 3 = c$$

$$5 = c$$

$$\boxed{f(x) = x^3 - 2x^2 + 5}$$

9. Find the specific expression of $f(x)$ if $f''(x) = 36x^2 - 6$, $f'(-1) = 3$, and $f(1) = 9$.

$$f'(x) = \frac{36x^3}{3} - 6x + c = 12x^3 - 6x + c$$

$$f'(-1) = 12(-1)^3 - 6(-1) + c$$

$$3 = -12 + 6 + c$$

$$3 = -6 + c$$

$$9 = c$$

$$f'(x) = 12x^3 - 6x + 9$$

$$f(x) = \frac{12x^4}{4} - \frac{6x^2}{2} + 9x + k$$

$$f(x) = 3x^4 - 3x^2 + 9x + k$$

$$f(1) = 3(1)^4 - 3(1)^2 + 9(1) + k$$

$$9 = 3 - 3 + 9 + k$$

$$9 = 0 + 9 + k$$

$$0 = k$$

$$\boxed{f(x) = 4x^3 - 3x^2 + 9x}$$