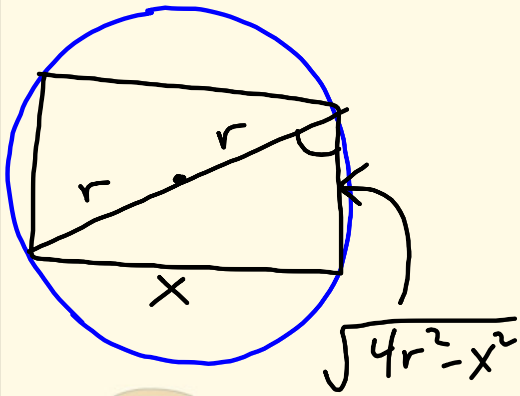
WARM UP!!

Find the area of the largest rectangle that can be inscribed in a circle of radius *r*.



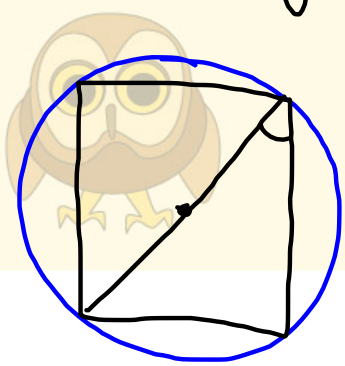
$$A = x\sqrt{4r^2-x^2}$$

$$\frac{dA}{dx} = \sqrt{4r^2-x^2} + x\left(\cancel{\tfrac{1}{2}}\right)\left(4r^2-x^2\right)^{-\frac{1}{2}}\left(-\cancel{2}x\right)$$

$$= \sqrt{4r^2-x^2} - \frac{x^2}{\sqrt{4r^2-x^2}}$$

$$= \frac{4r^2-x^2-x^2}{\sqrt{4r^2-x^2}}$$

$$= \frac{4r^2-2x^2}{\sqrt{4r^2-x^2}}$$



$$4r^2 = 2x^2$$

$$x^2 = 2r^2$$

$$\text{width} = x = \sqrt{2r^2} = r\sqrt{2}$$

$$\text{height} = \sqrt{4r^2-(r\sqrt{2})^2}$$

$$= \sqrt{4r^2-2r^2} = \sqrt{2r^2}$$

$$= r\sqrt{2}$$

(17) $\dfrac{dQ}{dx} = Kx(Q_0 - x)$ (K and Q_0 are constants)

$$\frac{d^2Q}{dx^2} = K(Q_0 - x) + Kx(-1)$$

$$= KQ_0 - Kx - Kx = 0$$

$$KQ_0 - 2Kx = 0$$

$$Q_0 - 2x = 0$$

$$2x = Q_0$$

$$x = \frac{Q_0}{2}$$

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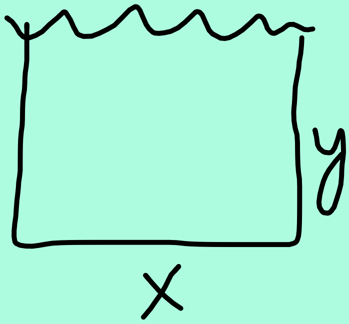


$$P = x + 2y$$

$$P = x + 2\left(\frac{180000}{x}\right)$$

$$A = xy = 180000$$

$$P = x + 360000x^{-1}$$

$$y = \frac{180000}{x}$$

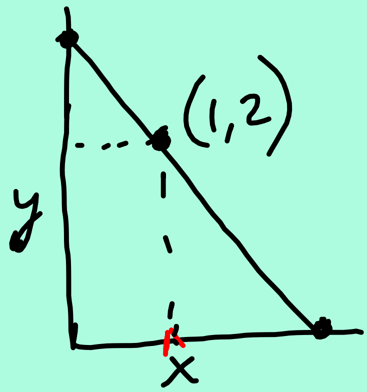
$$\frac{dP}{dx} = 1 - \frac{360000}{x^2}$$

$$\frac{x^2 - 360000}{x^2} = 0$$

$$x^2 = 360000$$

$$x = 600\ \text{m}, \quad y = \frac{1800\cancel{0}\cancel{0}}{6\cancel{0}\cancel{0}} = 300\ \text{m}$$

25



$$\frac{2-y}{1} = \frac{-2}{x-1}$$

$$-y = \frac{-2}{x-1} - 2$$

$$y = \frac{2}{x-1} + 2$$

$$y = \frac{2+2x-2}{x-1} = \frac{2x}{x-1}$$

a) $H = \sqrt{x^2+y^2}$

$$H = \sqrt{x^2+\left(\frac{2x}{x-1}\right)^2}$$

c) $A = \frac{1}{2}xy$

$$A = \cancel{\frac{1}{2}}x\left(\frac{\cancel{2}x}{x-1}\right)$$

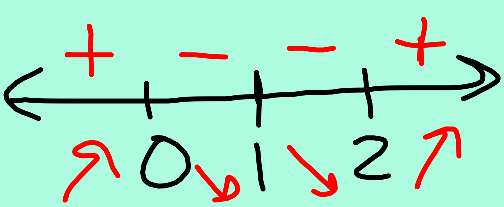
$$A = \frac{x^2}{x-1}$$

$$\frac{dA}{dx} = \frac{2x(x-1)-x^2(1)}{(x-1)^2}$$

$$= \frac{2x^2-2x-x^2}{(x-1)^2}$$

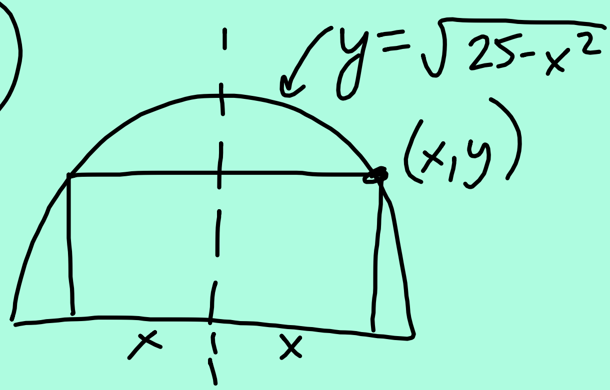
$$= \frac{x^2-2x}{(x-1)^2}$$

$$= \frac{x(x-2)}{(x-1)^2}$$



$$x = 2,\ y = \frac{4}{2-1} = 4$$

27



$$A = 2xy$$

$$A = 2x\sqrt{25-x^2}$$

$$\frac{dA}{dx} = 2\sqrt{25-x^2} + \cancel{2}x\left(\cancel{\tfrac{1}{2}}\right)(25-x^2)^{-\frac{1}{2}}(-2x)$$

$$= 2\sqrt{25-x^2} - \frac{2x^2}{\sqrt{25-x^2}}$$

$$\frac{2(25-x^2)-2x^2}{\sqrt{25-x^2}} = 0$$

$$\frac{50-4x^2}{\sqrt{25-x^2}} = 0$$

$$x = \sqrt{\frac{50}{4}} = \frac{5\sqrt{2}}{2} \qquad y = \sqrt{25-\frac{50}{4}}$$

$$= \sqrt{\frac{50}{4}} = \frac{5\sqrt{2}}{2}$$

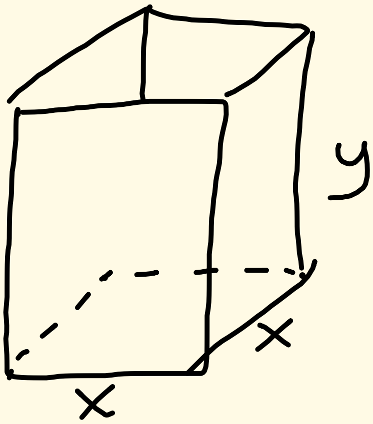
length $= 5\sqrt{2}$ width $= \frac{5\sqrt{2}}{2}$

3.7b Optimization Again!

At the end of this lesson you will be able to:

- Use derivatives to solve even more applications of max/min problems!

ex) A manufacturer wants to design an open box (no top) with a square base using 108 square inches of cardboard. What dimensions will produce a box of maximum volume?



$$V = x^2 y$$

$$V = x^{\cancel{2}}\left(\frac{108 - x^2}{4\cancel{x}}\right)$$

$$108 = x^2 + 4xy$$

$$y = \frac{108 - x^2}{4x}$$

$$V = 27x - \frac{1}{4}x^3$$

$$\frac{dV}{dx} = 27 - \frac{3}{4}x^2 = 0$$

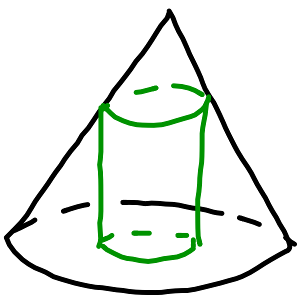
$$x^2 = \frac{27 \cdot 4}{3} = 36$$

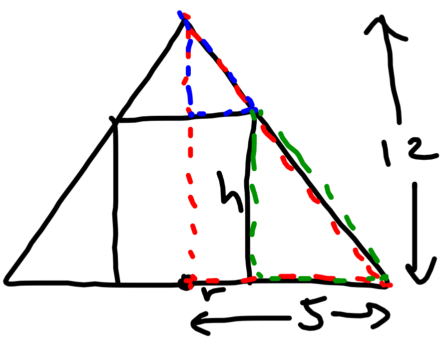
$$x = 6, \quad y = \frac{108 - 36}{24}$$

$$= 3$$

$6 \times 6 \times 3$ in

ex) Find the dimensions of the right circular cylinder of greatest volume that can be inscribed in a right circular cone with a radius of 5 cm and a height of 12 cm.





$$V = \pi r^2 h$$

$$V = \pi r^2\left(12 - \frac{12}{5}r\right)$$

$$V = 12\pi r^2 - \frac{12}{5}\pi r^3$$

$$\frac{dV}{dr} = 24\cancel{\pi} r - \frac{36}{5}\cancel{\pi} r^2 = 0$$

$$12r\left(2 - \frac{3}{5}r\right) = 0$$

$$r = 0 \quad r = \frac{10}{3}$$

$$h = 12 - \frac{\cancelto{4}{12}}{\cancel{5}}\cdot\frac{\cancelto{2}{10}}{\cancel{3}}$$

$$= 4$$

$$\frac{5}{12} = \frac{5-r}{h}$$

$$h = \frac{12(5-r)}{5} = 12 - \frac{12}{5}r$$

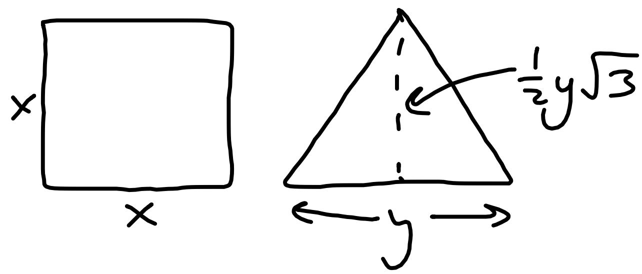
$$\boxed{r = \frac{10}{3},\ h = 4}$$

You try! Twenty feet of wire is to be used to form two figures. In each of the following cases, how much wire should be used for each figure so that the total area enclosed is a maximum?

a) square and equilateral triangle

b) square and circle

a



$$4x+3y=20$$
$$4x=20-3y$$
$$x=5-\frac{3}{4}y$$

$$A=x^2+\frac{1}{2}y\left(\frac{1}{2}y\sqrt{3}\right)$$
$$A=x^2+\frac{1}{4}y^2\sqrt{3}$$
$$A=\left(5-\frac{3}{4}y\right)^2+\frac{1}{4}y^2\sqrt{3}$$
$$\frac{dA}{dy}=\cancel{2}\left(5-\frac{3}{4}y\right)\left(-\frac{3}{\cancelto{2}{4}}\right)+\frac{1}{2}y\sqrt{3}$$
$$=-\frac{15}{2}+\frac{9}{8}y+\frac{1}{2}y\sqrt{3}=0$$
$$\frac{9y+4y\sqrt{3}}{8}=\frac{15}{2}$$
$$9y+4y\sqrt{3}=60$$
$$y=\frac{60}{9+4\sqrt{3}} \qquad x=5-\frac{3}{4}\left(\frac{60}{9+4\sqrt{3}}\right)$$

What have we learned??

- Can I use derivatives and the concepts of max/min to solve even more applications involving optimization?