WARM UP!!

Find 2 positive numbers whose product is 192 and whose sum is a minimum.

primary equation

$$S = x + y$$

secondary eq

$$xy = 192$$

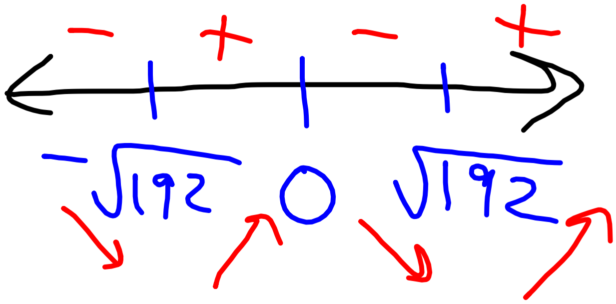
$$x = \frac{192}{y}$$

$$S = \frac{192}{y} + y$$

$$\frac{dS}{dy} = \frac{-192}{y^2} + 1$$

$$= \frac{-192 + y^2}{y^2}$$

$$y = \pm\sqrt{192}$$



$$y = \sqrt{192}$$

$$x = \sqrt{192}$$

39. $y = \sin x - \frac{1}{18}\sin 3x \quad [0, 2\pi]$

$y' = \cos x - \frac{1}{6}\cos 3x = 0$

$x \approx 1.5708,\ 4.7124$

y': $+ \quad - \quad +$

$0 \quad 1.5708 \quad 4.7124 \quad 2\pi$

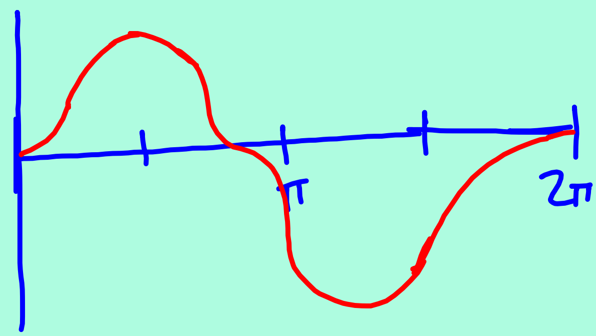
$\nearrow \quad \searrow \quad \nearrow$

$y'' = -\sin x + \frac{1}{2}\sin 3x$

$x \approx 0,$

$+ \quad - \quad + \quad - \quad + \quad -$

$0 \quad\quad 2\pi$



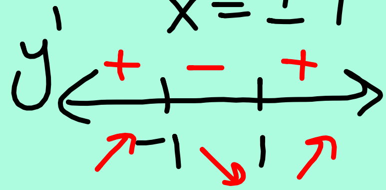
31 $y = x^5 - 5x$

$y' = 5x^4 - 5$

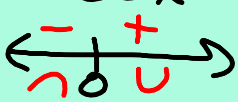
$5(x^4 - 1) = 0$

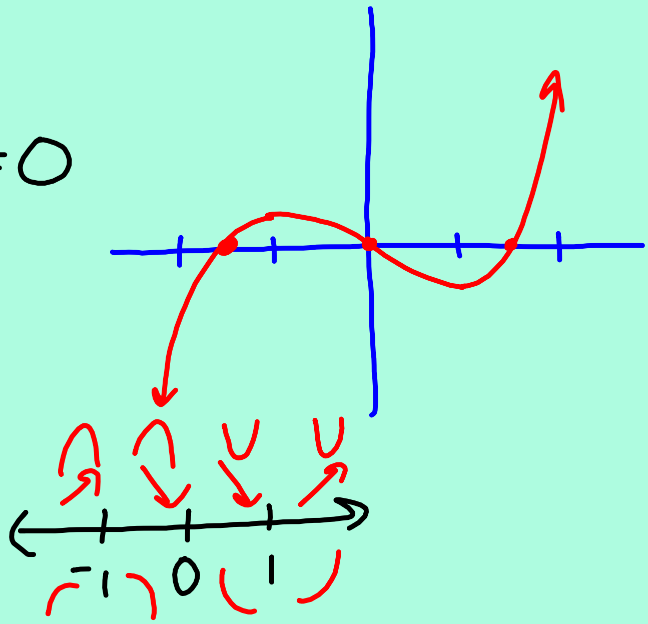
$x = \pm 1$

y'



$y'' = 20x^3$





~~45 $g(x) = x\tan x$ $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$~~

~~x-int: $x = 0, -\pi, \pi$~~

~~$g' = \tan x + x\sec^2 x$~~

~~$= \tan x + x(\tan^2 x - 1)$~~

~~$= \tan x + x\tan^2 x - x$~~

~~$\sin x\cos x + x = 0$~~

(17) $y = \dfrac{x^2-6x+12}{x-4}$

$$\begin{array}{r|rrr} 4 & 1 & -6 & 12 \\ & & 4 & -8 \\ \hline & 1 & -2 & 4 \end{array}$$

$y = x - 2$

$$y' = \frac{(2x-6)(x-4)-(x^2-6x+12)}{(x-4)^2}$$

$$= \frac{2x^2-14x+24-x^2+6x-12}{(x-4)^2}$$

$$= \frac{x^2-8x+12}{(x-4)^2}$$

$$= \frac{(x-2)(x-6)}{(x-4)^2}$$

y': $+$ | 2 | $-$ | 4 | $-$ | 6 | $+$

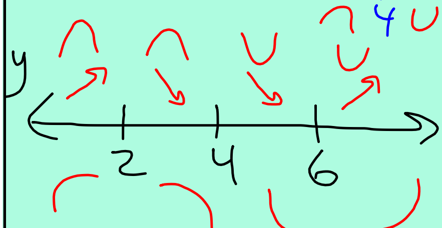
$$y'' = \frac{(2x-8)(x-4)^{\cancel{2}} - (x^2-8x+12)(2)\cancel{(x-4)}}{(x-4)^{\cancel{4}\,3}}$$

$$= \frac{\cancel{2x^2}-\cancel{16x}+32-\cancel{2x^2}+\cancel{16x}-24}{(x-4)^3}$$

$$= \frac{8}{(x-4)^3}$$

y'': $-$ | 4 | $+$

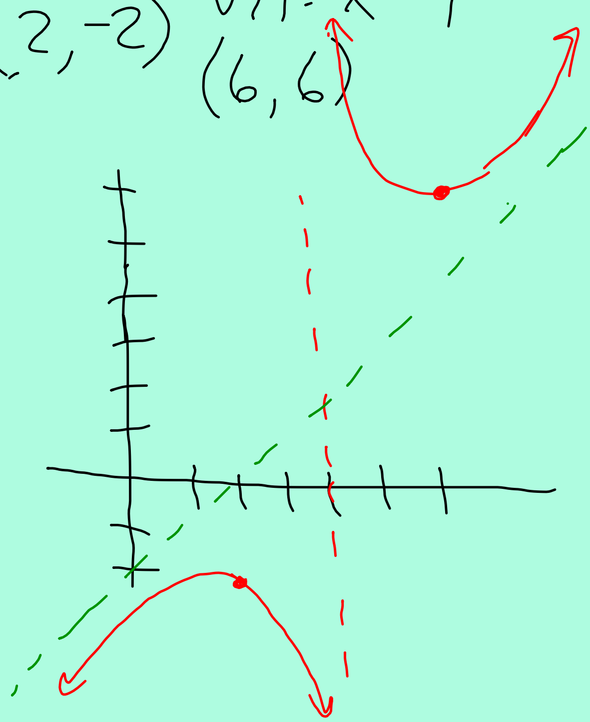
y



$(2,-2)$

VA: $x = 4$

$(6,6)$

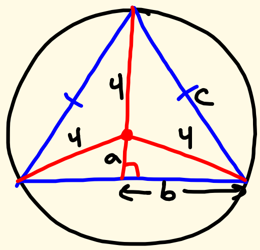


3.7 Optimization!

At the end of this lesson you will be able to:

- Use derivatives to solve applications of max/min problems

ex) Find the base and height of the largest isosceles triangle that can be inscribed in a circle of radius 4.



$$A = \frac{1}{2}(\text{base})(\text{height})$$

$$A = \cancel{\tfrac{1}{2}}(\cancel{2}b)(4+a)$$

$$A = \sqrt{16-a^2}\,(4+a)$$

$$a^2+b^2=4^2$$

$$b^2=16-a^2$$

$$b=\sqrt{16-a^2}$$

$$\frac{dA}{da} = \cancel{\tfrac{1}{2}}(16-a^2)^{-\frac{1}{2}}(-\cancel{2}a)(4+a)+\sqrt{16-a^2}$$

$$= \frac{-4a-a^2}{\sqrt{16-a^2}}+\sqrt{16-a^2}$$

$$= \frac{-4a-a^2+16-a^2}{\sqrt{16-a^2}}$$

$$= \frac{-2a^2-4a+16}{\sqrt{16-a^2}}$$

$$= \frac{-2(a^2+2a-8)}{\sqrt{16-a^2}}$$

$$= \frac{-2(a+4)(a-2)}{\sqrt{16-a^2}}$$

$$a=-4,\ 2$$

$$\text{height} = 4+2 = \boxed{6}$$

$$\text{base} = 2\sqrt{16-4} = 2\sqrt{12} = \boxed{4\sqrt{3}}$$

STEPS:

1) Draw a picture and label it

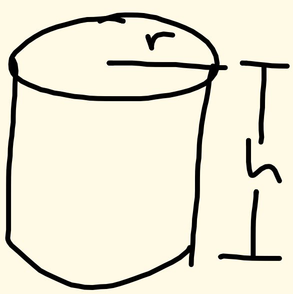
2) Write the primary equation - the equation for whatever you want to optimize

3) If the primary equation has more than one independent variable, write any secondary equations and substitute

4) Differentiate with respect to the independent variable

5) set derivative = 0 and solve

You try!) The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?



$$S = 2\pi r^2 + 2\pi r h$$

$$S = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$$

$$S = 2\pi r^2 + \frac{32\pi}{r}$$

$$V = 16\cancel{\pi} = \cancel{\pi} r^2 h$$

$$\frac{dS}{dr} = 4\pi r - \frac{32\pi}{r^2}$$

$$= \frac{4\pi r^3 - 32\pi}{r^2}$$

$$= \frac{4\pi(r^3 - 8)}{r^2}$$

$r = 2, 0$

$- \quad - \quad +$

$0 \quad 2$

$$h = \frac{16}{r^2} = \frac{16}{4} = 4\text{ in}$$

Which point(s) on the graph of y = x^2+1 is closest to the point (0, 4)?

$$x^2 = y - 1$$

$$D = \sqrt{(x-0)^2 + (y-4)^2}$$

$$D = \sqrt{y - 1 + (y-4)^2}$$

$$F = y - 1 + (y-4)^2$$

$$F = y - 1 + y^2 - 8y + 16$$

$$F = y^2 - 7y + 15$$

$$\frac{dF}{dy} = 2y - 7$$

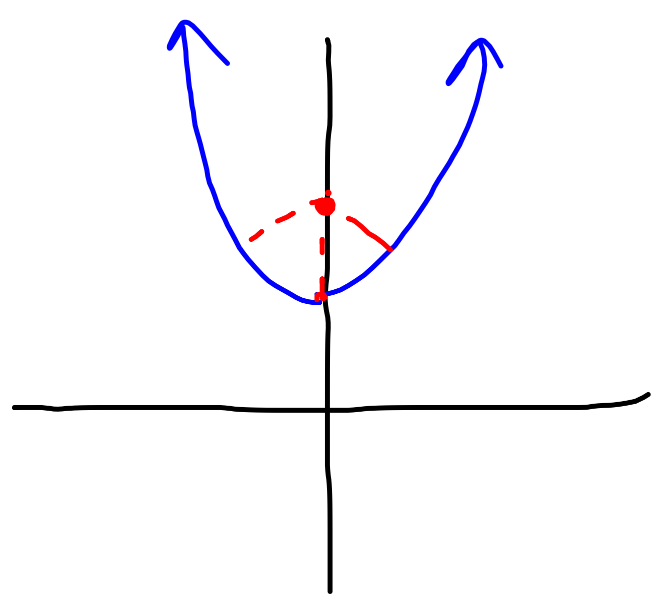
$$y = \frac{7}{2}$$

$$x^2 = \frac{7}{2} - 1$$

$$x^2 = \frac{5}{2}$$

$$x = \pm\sqrt{\frac{5}{2}}$$

$$\left(\sqrt{\frac{5}{2}}, \frac{7}{2}\right), \left(-\sqrt{\frac{5}{2}}, \frac{7}{2}\right)$$



What have we learned??

- Can I use derivatives and the concepts of max/min to solve applications involving optimization?