

$$(3) \quad f(x) = \frac{24}{x^2+12} = 24(x^2+12)^{-1}$$

$$f'(x) = -24(x^2+12)^{-2}(2x)$$

$$= \frac{-48x}{(x^2+12)^2}$$

$$f''(x) = \frac{-48(x^2+12)^2 - 48x(2)(x^2+12)(2x)}{(x^2+12)^4}$$

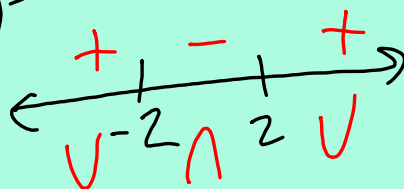
$$= \frac{-48(x^2+12) + 192x^2}{(x^2+12)^3}$$

$$= \frac{-48x^2 - 576 + 192x^2}{(x^2+12)^3}$$

$$= \frac{144x^2 - 576}{(x^2+12)^3}$$

$$x^2 = \frac{576}{144} = 4$$

$$x = \pm 2$$



$f$  is  $\cup$  on  $(-\infty, -2) \cup (2, \infty)$  b/c  $f'' > 0$

$f$  is  $\cap$  on  $(-2, 2)$  b/c  $f'' < 0$

$f$  is  $\cap$  @  $(-2, \frac{3}{2}), (2, \frac{3}{2})$  b/c

$f''$  chgs sign

$$\textcircled{9} \quad y = 2x - \tan x \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$y' = 2 - \sec^2 x$$

$$y'' = -2\sec x \cdot \sec x \tan x$$

$$= -2\underline{\sec^2 x} \tan x$$

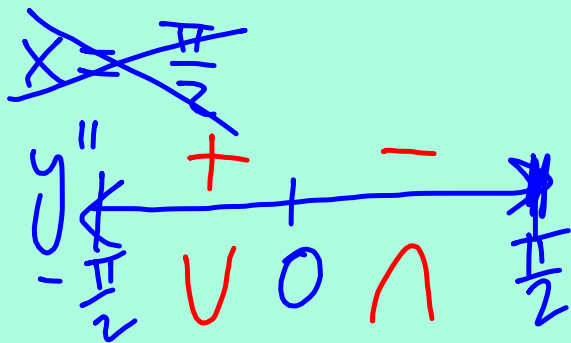
~~$$\sec x = 0$$~~

$$\cos x = 0$$

$$\tan x = 0$$

$$\sin x = 0$$

$$x = 0$$



$y$  is  $U$  on  $(-\frac{\pi}{2}, 0)$  b/c  $y'' > 0$

$y$  is  $n$  on  $(0, \frac{\pi}{2})$  b/c  $y'' < 0$

$$(15) \quad f(x) = x(x-4)^3$$

$$f'(x) = (x-4)^3 + x(3)(x-4)^2$$

$$= (x-4)^2(x-4+3x)$$

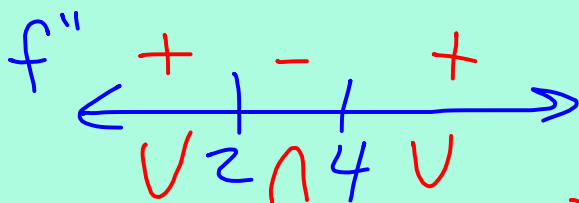
$$= (x-4)^2(4x-4)$$

$$f''(x) = 2(x-4)(4x-4) + (x-4)^2(4)$$

$$= 4(x-4)[2(x-1) + x-4]$$

$$= 4(x-4)(3x-6)$$

$$x = 4, 2$$



$f$  is  $\cup$  on  $(-\infty, 2) \cup (4, \infty)$  b/c  $f'' > 0$

$f$  is  $\cap$  on  $(2, 4)$  b/c  $f'' < 0$

$p^{\circ}$ : @  $(2, -16), (4, 0)$  b/c

$f''$  chgs sign

Homework 3-4a

$$(21) f(x) = \sin\left(\frac{x}{2}\right) \quad [0, 4\pi]$$

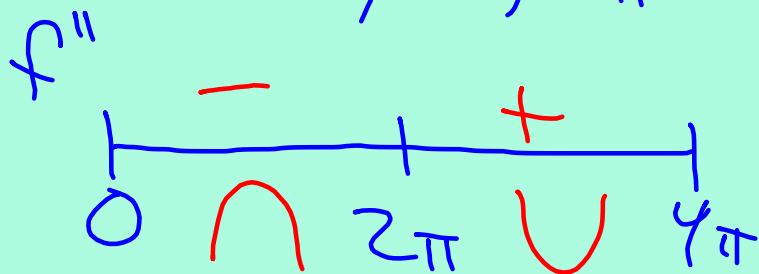
$$f'(x) = \frac{1}{2} \cos\left(\frac{x}{2}\right)$$

$$f''(x) = -\frac{1}{4} \sin\left(\frac{x}{2}\right) = 0$$

$$\sin\left(\frac{x}{2}\right) = 0$$

$$\frac{x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, \dots$$

$$x = 0, 2\pi, 4\pi$$



$f$  is  $\cap$  on  $(0, 2\pi)$  b/c  $f'' < 0$

$f$  is  $\cup$  on  $(2\pi, 4\pi)$  b/c  $f'' > 0$

poi @  $(2\pi, 0)$  b/c  $f''$  chgs sign

$$(23) \quad f(x) = \sec\left(x - \frac{\pi}{2}\right) \quad (0, 4\pi)$$

$$= \csc x$$

Let's do it with secant just for fun!

$$f'(x) = \sec\left(x - \frac{\pi}{2}\right) \tan\left(x - \frac{\pi}{2}\right)$$

$$\begin{aligned} f''(x) &= \sec\left(x - \frac{\pi}{2}\right) \tan\left(x - \frac{\pi}{2}\right) \tan\left(x - \frac{\pi}{2}\right) \\ &\quad + \sec\left(x - \frac{\pi}{2}\right) \sec^2\left(x - \frac{\pi}{2}\right) \\ &= \sec\left(x - \frac{\pi}{2}\right) \tan^2\left(x - \frac{\pi}{2}\right) + \sec^3\left(x - \frac{\pi}{2}\right) \\ &= \sec\left(x - \frac{\pi}{2}\right) \left[ \tan^2\left(x - \frac{\pi}{2}\right) + \sec^2\left(x - \frac{\pi}{2}\right) \right] \\ &= \sec\left(x - \frac{\pi}{2}\right) \left[ 2\sec^2\left(x - \frac{\pi}{2}\right) - 1 \right] \end{aligned}$$

$$\sec\left(x - \frac{\pi}{2}\right) = 0$$

$$\frac{\text{und}}{\cos\left(x - \frac{\pi}{2}\right)} = 0$$

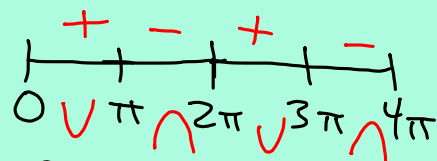
$$x - \frac{\pi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

$$x = \pi, 2\pi, 3\pi$$

$$\sec^2\left(x - \frac{\pi}{2}\right) = \frac{1}{2}$$

$$\sec\left(x - \frac{\pi}{2}\right) = \pm \frac{1}{\sqrt{2}}$$

$$\cos\left(x - \frac{\pi}{2}\right) = \pm \sqrt{2}$$



$f$  is  $\cup$  on  $(0, \pi) \cup (2\pi, 3\pi)$  b/c  $f'' > 0$

$f$  is  $\cap$  on  $(\pi, 2\pi) \cup (3\pi, 4\pi)$  b/c  $f'' < 0$

no poi (asymptotes)

$$(25) \quad f(x) = 2\sin x + \sin 2x \quad [0, 2\pi]$$

$$f'(x) = 2\cos x + 2\cos 2x$$

$$f''(x) = -2\sin x - 4\sin 2x$$

$$= -2\sin x - 8\sin x \cos x$$

$$= -2\sin x(1 + 4\cos x) = 0$$

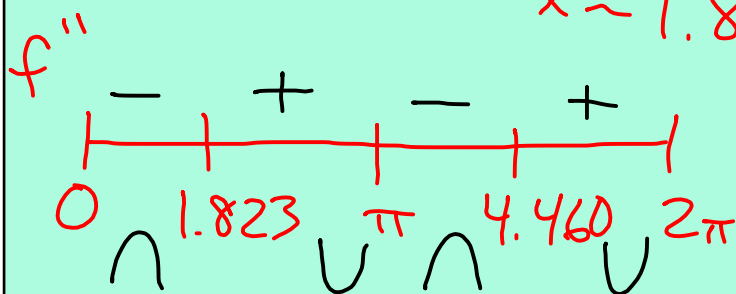
$$\sin x = 0$$

$$x = 0, \pi, 2\pi$$

$$\cos x = -\frac{1}{4}$$

$$x = \arccos\left(-\frac{1}{4}\right)$$

$$x \approx 1.823, 4.46$$



$f$  is  $\cup$  on  $(1.823, \pi) \cup (4.460, 2\pi)$  b/c  $f'' > 0$

$f$  is  $\cap$  on  $(0, 1.823) \cup (\pi, 4.460)$  b/c  $f'' < 0$

poi @  $(1.823, \quad), (\pi, \quad)$   
 $(4.460, \quad)$  b/c  $f''$  chgs sign