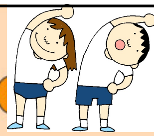
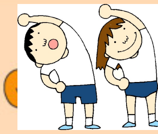
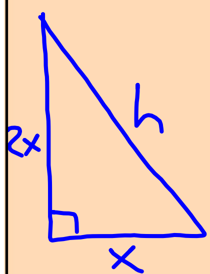
 

WARM UP!!

One leg of a right triangle is twice the length of the other. If the hypotenuse is growing at a rate of 3 in/sec, how fast is the area of the triangle growing when the hypotenuse is 10 in?



$\frac{dh}{dt} = 3$

$\frac{dA}{dt} = ?$

$h = 10$

$A = \frac{1}{2}(x)(2x)$

$A = x^2$

$\frac{dA}{dt} = 2x\frac{dx}{dt}$

$= 2(2\sqrt{5})\left(\frac{3}{\sqrt{5}}\right)$

$= 12$

$x^2 + (2x)^2 = h^2$

$5x^2 = h^2$

$10x\frac{dx}{dt} = 2h\frac{dh}{dt}$

$100 = 5x^2$

$x = 2\sqrt{5}$

$10(2\sqrt{5})\frac{dx}{dt} = 2(10)(3)$

$\frac{dx}{dt} = \frac{3}{\sqrt{5}}$

OR!

$A = \frac{1}{5}h^2$

$\frac{dA}{dt} = \frac{2}{5}h\frac{dh}{dt}$

$= \frac{2}{5}(10)(3) = 12$

12 in²/sec ✓

3.1 Critical Values and Absolute Extrema

At the end of this lesson you will be able to:

- Locate, analytically and graphically, all critical values (a.k.a. critical numbers) on a function
- Locate and justify the absolute max/min values for a function

Historical note

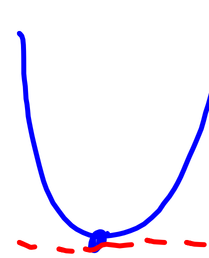
Much of the work in this unit is based on the work and theorems of **Pierre de Fermat**, who lived in France in the early 1600's (same time as Rene Descartes). Many of the advancements in calculus discovered and published by Isaac Newton and Gottfried Leibniz were based on his work. His famous 'last theorem' (found written in the margin of a book by Diophantus) was not proven until 1994!

2nd Warmup) Joe started mowing lawns for his neighbors his freshman year of high school to earn some extra money. His annual earnings from 2010 to 2013 could be modeled by the function $E(t) = 3t^2 - 6t + 4$, where $t = 0$ represents the end of 2010 and $E(t)$ represents his total earnings for each year in hundreds of dollars.

What were Joe's lowest earnings and what were his highest earnings? When did these occur?

lowest \$100 in 2011

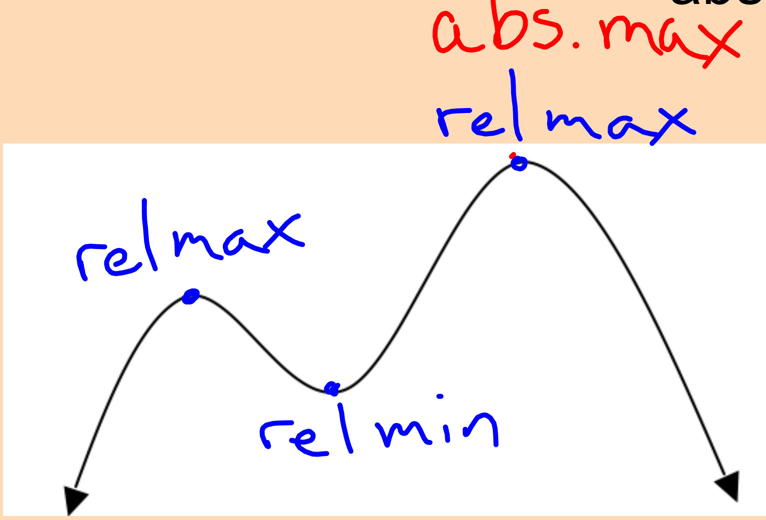
highest \$1300 in 2013



extrema

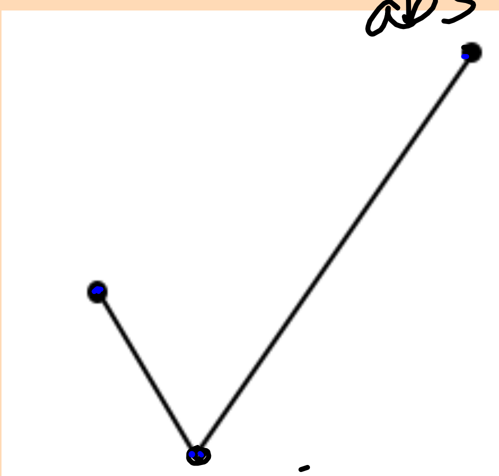
Two types of maxima/minima: relative (local)

absolute

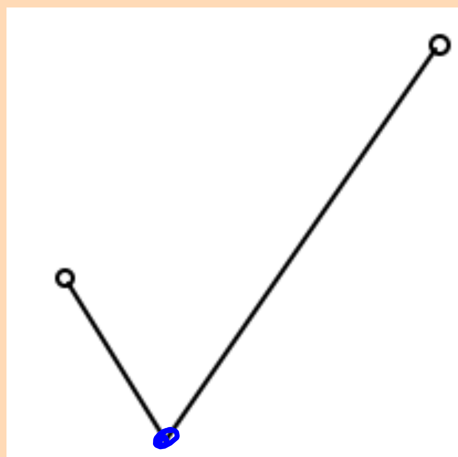


no abs. min.

abs max



rel min
abs min



rel min
abs min

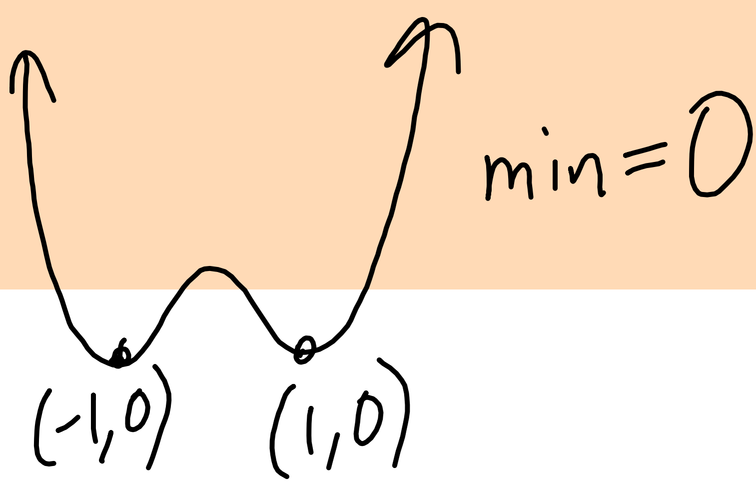
no abs max

Absolute max = the highest y-value on a function

Absolute min = the lowest y-value on a function

So if the absolute maximum occurs at the point (5, -3), the maximum value for the function is -3 and it occurs at x = 5.

(think in context)



A FEW GOOD THEOREMS AND A DEFINITION

1. EXTREME VALUE THEOREM: If f (x) is continuous on [a, b], then it MUST have both a maximum and a minimum on [a, b]

2. CRITICAL VALUE (or critical number) = an x-value, in the domain of a function, at which the derivative is either 0 or undefined

3. LOCATION OF EXTREMA: If f (x) is continuous on [a, b], then the absolute extrema MUST occur at either a critical number or at an endpoint

4. RELATIVE EXTREMA (attributed to Fermat): all relative extrema must occur at a critical value

(NOTE: but not every critical value yields a relative extremum)

STEPS!! If ALL absolute extrema must be located at either a critical value or at an endpoint, what would the steps be to locate these and justify your answer(s)?

1. Find the derivative

2. Find the critical numbers

3. TEST the critical numbers and endpoints by 'plugging' them into the... **original function**

4. Write your answer(s), remember that absolute extrema are **y-values**, not ordered pairs

YOU TRY!

1. Find all critical numbers for $f(x) = (x - 1)^{2/3}$.
Then locate the abs max/min on [-1, 2]. max $= \sqrt[3]{4}$
min $= 0$

$f'(x) = \frac{2}{3}(x-1)^{-\frac{1}{3}}$

$= \frac{2}{3\sqrt[3]{x-1}}$

$3\sqrt[3]{x-1} = 0$

$\sqrt[3]{x-1} = 0$

$x-1=0$

crit#
$x = 1$

Test

$f(-1) = \sqrt[3]{(-1-1)^2} = \sqrt[3]{4}$

$f(1) = \sqrt[3]{(1-1)^2} = 0$

$f(2) = \sqrt[3]{(2-1)^2} = 1$

(If you finish #1 and want to take it up a notch, try this one!)

2. Find all critical numbers for $f(x) = 4/3x\sqrt{3 - x}$.
Then locate the abs max/min on [0, 3].

$f(x) = \frac{4}{3}x\sqrt{3-x}$

$f'(x) = \frac{4}{3}\sqrt{3-x} + \frac{4}{3}x\left(\frac{1}{2}\right)(3-x)^{-\frac{1}{2}}(-1)$

$= \frac{4\sqrt{3-x}}{3} - \frac{2x}{3\sqrt{3-x}}$

$= \frac{4(3-x)-2x}{3\sqrt{3-x}} = \frac{12-6x}{3\sqrt{3-x}}$

crit#s: $x = 2, 3$

$f(0) = 0$

$f(2) = \frac{8}{3}$

$f(3) = 0$

max $= \frac{8}{3}$

min $= 0$

It's time for,

Fun With Calculators!!

Do the following on your calculator:

1. Enter $y = x^3 - 3x^2 + 3$ into Y1
2. Set the window to view x-values from -5 to 5
3. Use the calculator to find the max and min values for the function
4. Enter the derivative of the function in Y2 and graph it
5. Use the calculator to find the zeros of the derivative

Remember, a function that is increasing implies that a derivative is positive, not necessarily increasing

What have we learned?

- What is a critical number?
- Is an absolute max an x-value, a y-value, or an ordered pair?
- After finding the critical values, do I plug these numbers (and the endpoints) into the derivative or the original function to test for max/mins?

Fun derivative moment: