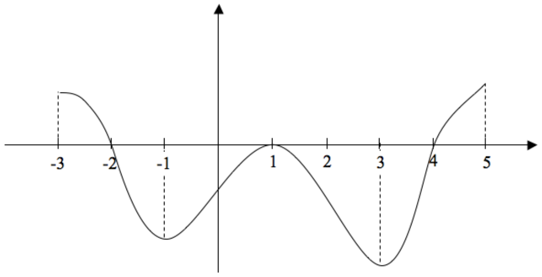
October 12, 2016

AP Exam Question #1 from 1996

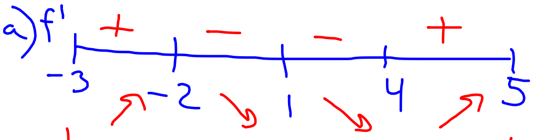


Note: This is the graph of the derivative of *f*, not the graph of *f*.

The figure above shows the graph of f', the derivative of a function *f*. The domain of *f* is the set of all real numbers x such that $-3 < x < 5$.

(a) For what values of x does *f* have a relative maximum? Why?
(b) For what values of x does *f* have a relative minimum? Why?
(c) On what intervals is the graph of *f* concave upward?
Use f' to justify your answer.
(e) Suppose that $f(1) = 0$. In the xy-plane provided, draw a sketch that shows the general shape of the graph of the function *f* on the open interval $0 < x < 2$.

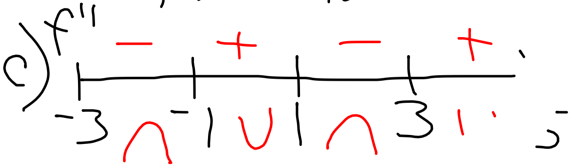
a)



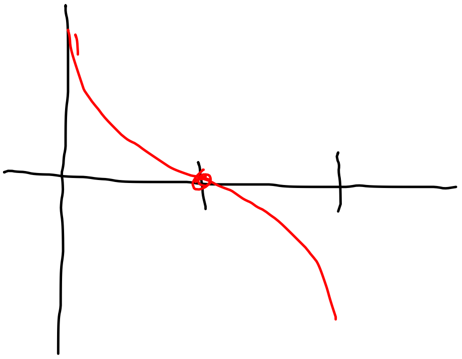
rel max @ x = -2 b/c f' chg. from + to −

b) rel min @ x=4 b/c f' ch from − to +

c)



f is c u on $(-1,1) \cup (3,5)$ b/c f' is inc.

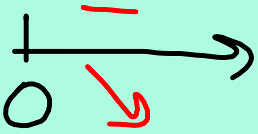


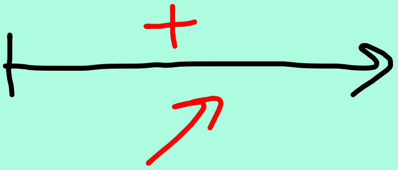
(HW) (63) $xy^2=4$ $y=\pm\frac{2}{\sqrt{x}}=\pm 2x^{-\frac{1}{2}}$

VA: $x=0$
HA: $y=0$

$y'=-x^{-\frac{3}{2}}=-\frac{1}{\sqrt{x^3}}$

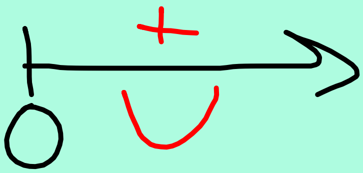
$y=x^{-\frac{3}{2}}=\frac{1}{\sqrt{x^3}}$

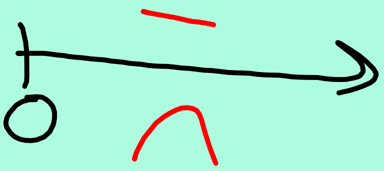


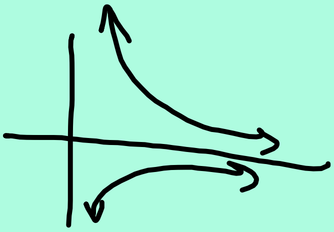


$y''=\frac{3}{2\sqrt{x^5}}$

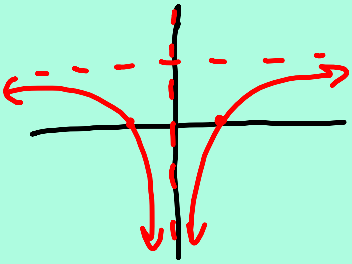
$y''=\frac{-3}{2\sqrt{x^5}}$







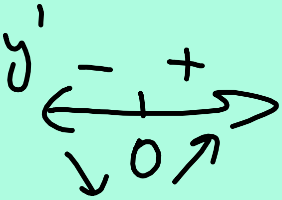
(67) $y = 2 - \frac{3}{x^2} = \frac{2x^2-3}{x^2}$



VA $x=0$

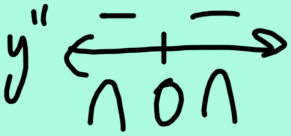
HA $y=2$

$$y' = \frac{4x(x^2) - (2x^2-3)(2x)}{x^4}$$



$$= \frac{4x^3 - 4x^3 + 6x}{x^4} = \frac{6x}{x^4} = \frac{6}{x^3}$$

$$y'' = \frac{-18}{x^4}$$



⑺1 $y=\frac{x^3}{\sqrt{x^2-4}}$

$D:(-\infty,-2]\cup[2,\infty)$

$VA: x=\pm 2$

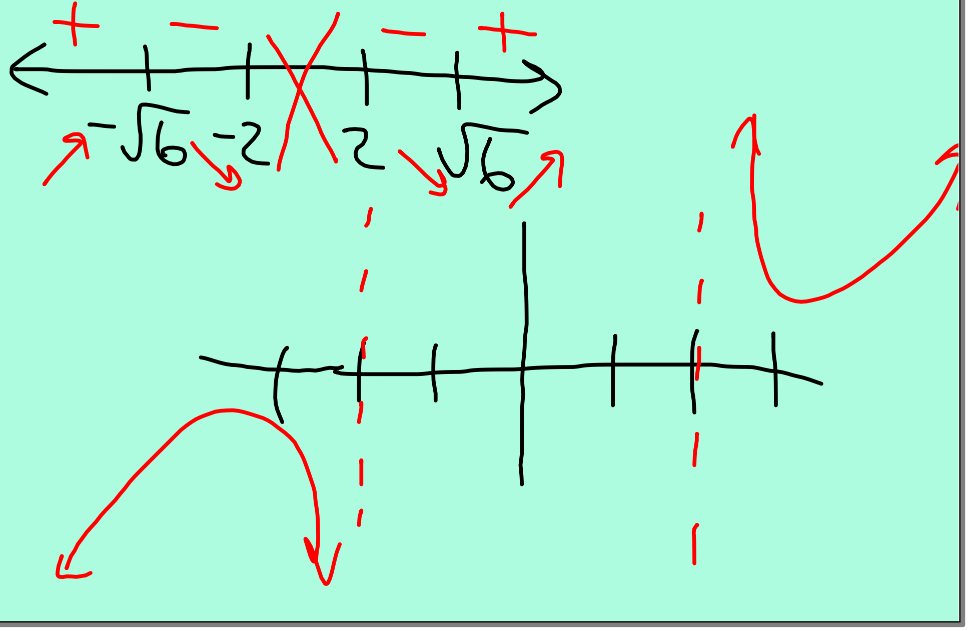
no HA^s

$$y'=\frac{3x^2\sqrt{x^2-4}-x^3\left(\frac{1}{\cancel{2}}\right)(x^2-4)^{-\frac{1}{2}}(\cancel{2}x)}{x^2-4}$$

$$=\frac{3x^2\sqrt{x^2-4}-\frac{x^4}{\sqrt{x^2-4}}}{x^2-4}$$

$$=\frac{3x^2(x^2-4)-x^4}{(x^2-4)\sqrt{x^2-4}}=\frac{2x^4-12x^2}{(x^2-4)\sqrt{x^2-4}}$$

$$=\frac{2x^2(x^2-6)}{(x^2-4)\sqrt{x^2-4}}$$ $x=0, \pm\sqrt{6},$ ± 2

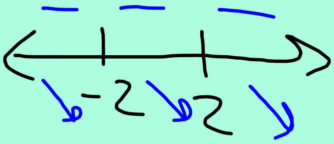


(57) $y = \frac{x}{x^2-4}$

VA: $x = \pm 2$

HA: $y = 0$

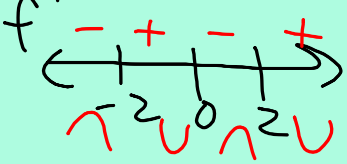
$$y' = \frac{x^2-4-x(2x)}{(x^2-4)^2} = \frac{\overset{-(x^2+4)}{-x^2-4}}{(x^2-4)^2}$$

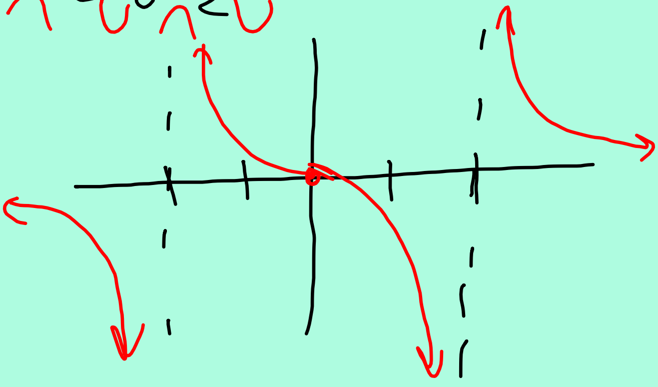


$$y'' = \frac{(-2x)(x^2-4)^{\cancel{2}} - (-x^2-4)(2)\cancel{(x^2-4)}(2x)}{(x^2-4)^{\cancel{4}\,3}}$$

$$= \frac{-2x^3+8x+4x^3+16x}{(x^2-4)^3}$$

$$= \frac{2x^3+24x}{(x^2-4)^3} = \frac{2x(x^2+12)}{(x^2-4)}$$





ⓧ 81 $g(x) = \sin\left(\frac{x}{x-2}\right), \quad x > 3$

no VA

no HA

$$g'(x) = \cos\left(\frac{x}{x-2}\right)\left(\frac{x-2-x}{(x-2)^2}\right)$$

$$= \cos\left(\frac{x}{x-2}\right)\left(\frac{-2}{(x-2)^2}\right)$$

$$\frac{x}{x-2} = \frac{\pi}{2} \qquad x = \frac{-2\pi}{2-\pi}$$

$$2x = \pi x - 2\pi$$

3.6 Graphing!!

ESSENTIAL LEARNING TARGET

At the end of this lesson you will be able to:

- identify key features of functions and their derivatives related to their graphical, numerical and analytical representations

Suppose $f(x) = \frac{2x^2}{9-x^2}$ $f'(x)=\frac{36x}{(9-x^2)^2}$ $f''(x)=\frac{108(x^2+3)}{(9-x^2)^3}$

Find the following:

a) domain	f) local maxima	k) intervals concave up
b) x-intercepts	g) local minima	l) intervals concave down
c) critical #s	h) abs maximum	m) points of inflection
d) intervals inc	i) abs minimum	n) vertical asymptotes
e) intervals dec	j) f"(x) = 0 or und	o) horizontal asymptotes

a) $\boxed{(-\infty,-3)\cup(-3,3)\cup(3,\infty)}$

b) $2x^2=0$ $\boxed{x=0}$

c) $\boxed{x=0}$ (±3 are not in domain)

d) f': − − + + at −3, 0, 3 (↘ −3 ↘ 0 ↗ 3 ↗)

$\boxed{f(x) \text{ is inc on } (0,3)\cup(3,\infty) \text{ b/c } f'(x)>0}$

e) $\boxed{f(x) \text{ is dec on } (-\infty,-3)\cup(-3,0) \text{ b/c } f'(x)<0}$

f) $\boxed{\text{no local max}}$

g) $\boxed{\text{local min @ } (0,0) \text{ b/c } f'(x) \text{ chgs from } (-) \text{ to } (+)}$

h) abs max = $\boxed{\text{none}}$

i) abs min = $\boxed{\text{none}}$

j) f'' is und @ $x=\pm3$

k) f'': − + − at −3, 3 (∩ −3 ∪ 3 ∩)

$\boxed{f \text{ is } \cup \text{ on } (-3,3) \text{ b/c } f''(x)>0}$

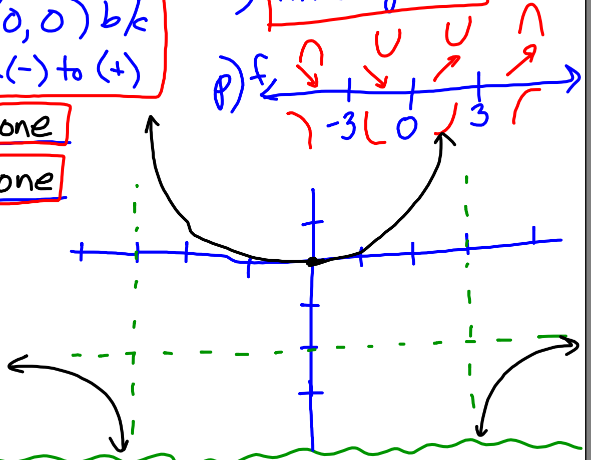
l) $\boxed{f \text{ is } \cap \text{ on } (-\infty,-3)\cup(3,\infty) \text{ b/c } f''(x)<0}$

m) $\boxed{\text{no p}^{\text{s}}\text{oi}}$

n) $\boxed{\text{VA}^{\text{s}} \text{ @ } x=3, x=-3}$

o) $\boxed{\text{HA @ } y=-2}$

p)



$$f'(x)=\frac{(9-x^2)(4x)-(2x^2)(-2x)}{(9-x^2)^2}=\frac{36x-4x^3+4x^3}{(9-x^2)^2}=\frac{36x}{(9-x^2)^2}$$

$$f''(x)=\frac{(9-x^2)^{\cancel{2}}(36)-(36x)(2)\cancel{(9-x^2)}(-2x)}{(9-x^2)^{\cancel{4}3}}=\frac{36(9-x^2+4x^2)}{(9-x^2)^3}$$

$$=\frac{36(3x^2+9)}{(9-x^2)^3}=\frac{108(x^2+3)}{(9-x^2)^3}$$

What have we learned??

- Can I use methods I learned in calculus to accurately sketch the graph of a rational function?