Let's Warm Up Those Brains...

For each of the theorems below, explain in your own words what the theorem says and draw a picture that represents the theorem (try to not look back at your notes)

*Intermediate Value Theorem

*Extreme Value Theorem

*Rolle's Theorem

*Mean Value Theorem

IVT

To get from f(a) to f(b) on a continuous function, you must pass through every y-value in between at least once

Conditions: cont on [a, b], k is between f(a) and f(b)

EVT

Every function that is continuous on a closed interval must have a max and min on that interval

Conditions: cont on [a, b]

$f(c) \geq f(x)$ for all x in $[a,b]$

Rolle's

Every function that is continuous and differentiable with endpoints at the same y-values must have a horizontal tangent inside the interval

Conditions: cont on [a, b], diff on (a, b), f(a) = f(b)

MVT

Every function that is continuous and differentiable must have at least one place in the interval where the slope of the tangent is equal to the slope of the secant line that connects the endpoints

Conditions: cont on [a, b], diff on (a, b)

3.5 Limits at Infinity!!

ESSENTIAL LEARNING TARGETS

At the end of this lesson you will be able to:

- explain asymptotic and unbounded behavior of functions using limits
- evaluates limits using comparative growth rates
- determines relative magnitudes of functions and their rates of change using limits

2nd warmup!

Find each of the following:

$$\lim_{x \to \infty} 5 - \frac{2}{x^2} \approx 5$$

$$\lim_{x \to -\infty} 5 - \frac{2}{x^2} \approx 5$$

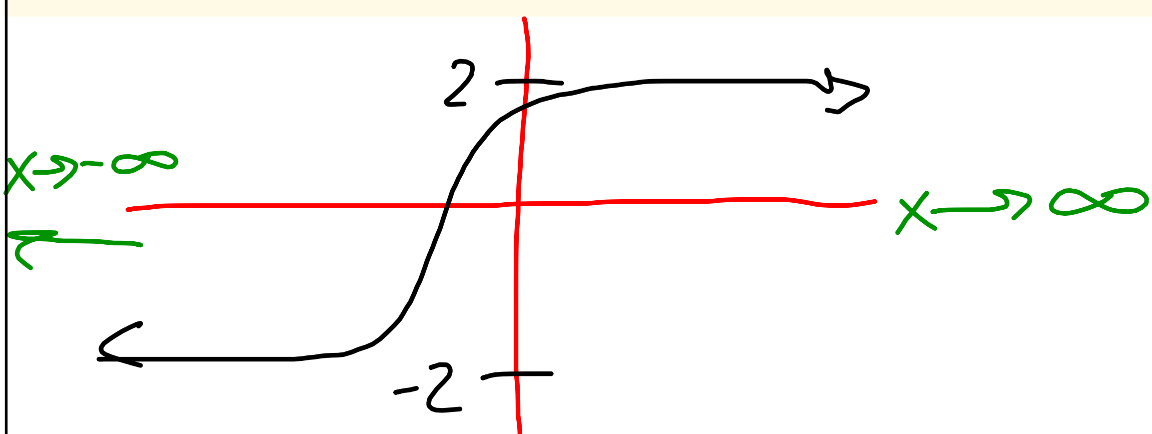
Simple horizontal asymptote rules

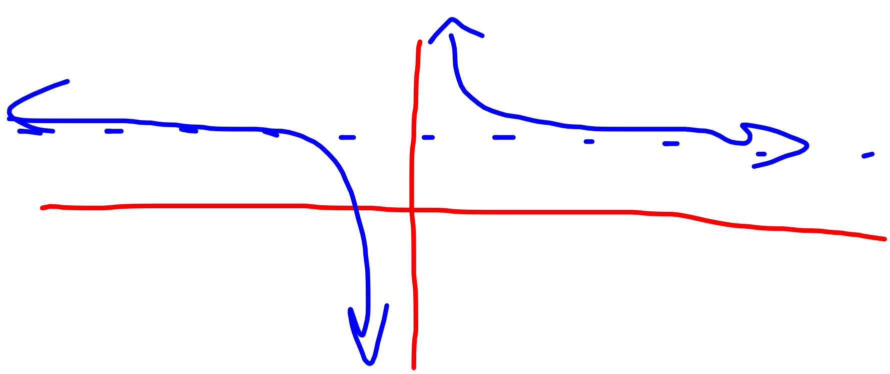
(for division of polynomials)

If $f(x) = p(x) / q(x)$, and if a = degree of $p(x)$ and b = degree of $q(x)$, then

- If $a < b$, then $y = 0$ is a horizontal asymptote
- If $a = b$, then y = leading coef of $p(x)$ / leading coef of $q(x)$
- If $a > b$, then there is no horizontal asymptote (so the limit will approach either ∞ or $-\infty$)

Note: the H.A. exists, then the limit as x approaches ∞ or $-\infty$ will equal the H.A.





One example, 2 methods:

1) Using H.A.s

$$\lim_{x\to\infty} \frac{8x^{2}+1}{9x^{2}-3x-5}$$

$$= \frac{8}{9}$$

2) divide top and bottom by highest degree of x

$$\lim_{x\to\infty} \frac{8x^2+1}{9x^2-3x-5} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{8+\frac{1}{x^2}}{9-\frac{3}{x}-\frac{5}{x^2}}$$

$$= \frac{8}{9}$$

Try these!

a) $\lim\limits_{x\to\infty} \dfrac{3x^2+1}{2x-5} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}} = \infty$

$= \lim\limits_{x\to\infty} \dfrac{3x+\frac{1}{x}}{2-\frac{5}{x}}$

b) $\lim\limits_{x\to-\infty} \dfrac{3x^2+1}{2x-5} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim\limits_{x\to-\infty} \dfrac{3x+\frac{1}{x}}{2-\frac{5}{x}} = -\infty$

c) $\lim\limits_{x\to\infty} \dfrac{3x+1}{5-2x}$

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d) $\lim\limits_{x\to-\infty} \dfrac{3x+1}{5-2x} = -\dfrac{3}{2}$

e) $\lim\limits_{x\to\infty} \dfrac{3x+1}{2x^2-5}$ 0

f) $\lim\limits_{x\to-\infty} \dfrac{3x+1}{2x^2-5} = 0$

That's great, but what if the function isn't a ratio of polynomials? Again, one example, 2 methods:

ex) Find the horizontal asymptotes for

1) simplify to leading terms

$$y=\frac{-2x+6}{\sqrt{5x^2+1}}$$

$$\lim_{x\to\infty}\frac{-2x+6}{\sqrt{5x^2+1}}$$

$$=\lim_{x\to\infty}\frac{-2x}{\sqrt{5x^2}}$$

$$=\lim_{x\to\infty}\frac{-2x}{\sqrt{5}\cdot x}=-\frac{2}{\sqrt{5}}$$

$$\lim_{x\to-\infty}\frac{-2x+6}{\sqrt{5x^2+1}}$$

$$=\lim_{x\to-\infty}\frac{-2x}{\sqrt{5x^2}}$$

$$=\lim_{x\to-\infty}\frac{-2x}{-\sqrt{5}x}=\frac{2}{\sqrt{5}}$$

$$\boxed{y=-\frac{2}{\sqrt{5}},\ \frac{2}{\sqrt{5}}}$$

because $\sqrt{x^2}=-x$ when $x<0$

2) divide top and bottom by highest degree of x

$$y=\frac{-2x+6}{\sqrt{5x^2+1}}$$

$$\lim_{x\to\infty}\frac{-2x+6}{\sqrt{5x^2+1}}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}\qquad \frac{1}{x}=\frac{1}{\sqrt{x^2}}$$

$$=\lim_{x\to\infty}\frac{-2+\frac{6}{x}}{\sqrt{5+\frac{1}{x^2}}}$$

$$=-\frac{2}{\sqrt{5}}$$

$$\lim_{x\to-\infty}\frac{-2x+6}{\sqrt{5x^2+1}}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}\qquad \frac{1}{x}=\frac{-1}{\sqrt{x^2}}$$

$$=\lim_{x\to-\infty}\frac{-2+\frac{6}{x}}{-\sqrt{5+\frac{1}{x^2}}}$$

$$=\frac{2}{\sqrt{5}}$$

Remember that y = sinx oscillates between -1 and 1

ex) $\lim_{x \to \infty} \frac{\sin x}{x} = 0$

You try!

① $\lim_{x \to \infty} \frac{3}{x} = 0$

② $\lim_{x \to \infty} \frac{5}{2x}$

③ $\lim_{x \to \infty} \frac{4}{7} = \frac{4}{7}$

④ $\lim_{x \to \infty} \frac{x}{3x}$

⑤ $\lim_{x \to \infty} \frac{2x}{x^2} = 0$

⑥ $\lim_{x \to \infty} \frac{23}{x^3}$

⑦ $\lim_{x \to \infty} \frac{3x}{2x+1} = \frac{3}{2}$

⑧ $\lim_{x \to -\infty} \frac{2x}{x^2}$

⑨ $\lim_{x \to -\infty} \frac{3x}{2x+1} = \frac{3}{2}$

⑩ $\lim_{x \to \infty} \frac{2x}{x^2}$

⑪ $\lim_{x \to \infty} \frac{6x^2-4}{7x^2+8} = \frac{6}{7}$

⑫ $\lim_{x \to -\infty} \frac{-2x^2-3x}{4x^2+8}$

⑬ $\lim_{x \to -\infty} \frac{x-3}{\sqrt{5x^2+6}} = \frac{-1}{\sqrt{5}}$

⑭ $\lim_{x \to \infty} \frac{2x-7}{\sqrt{4+7x^2}}$

Check your answers!

1) 0 2) 0 3) 4/7 4) 1/3 5) 0

6) 0 7) 3/2 8) 0 9) 3/2 10) 0

11) 6/7 12) -1/2 13) $-1/\sqrt{5}$ 14) $2/\sqrt{7}$

Comparative Growth Rates!

If you are evaluating the limit as x approaches infinity, and you have a different type of function in the numerator and denominator, often you can use general knowledge about each function's rate of growth to evaluate the limit

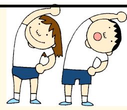
$$\lim_{x \to \infty} \frac{\text{faster}}{\text{slower}} = \pm\infty \qquad \lim_{x \to \infty} \frac{\text{slower}}{\text{faster}} = 0$$

logarithmic   polynomial/radical   exponential   factorial

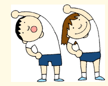
slower $\rightarrow$ faster

$$\lim_{x\to\infty} \frac{8x^{5000} + 99999999}{2^x} = 0$$

$$\lim_{x\to\infty} \frac{\sqrt{x} - 6}{\log_8 x} = \infty$$



WARM UP!!



1) Find a and b if f (x) is differentiable and

$$f(x) = \begin{cases} ax^2 + bx + 3, & -\infty < x \leq -2 \\ ax + b, & -2 < x < \infty \end{cases}$$

2) Given $y = 2x^3 - 9x^2$, find the critical values, relative extrema, points of inflection, and sketch the graph

3) A right cylinder's height is twice its radius. How fast is the cylinder's volume changing when its height is 4" and its radius is growing at 3" per second?

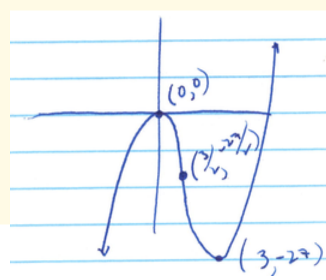
1) a = 1/3, b = 5/3

2) critical numbers @ x = 0, 3

rel max @ (0, 0) b/c y' chgs from (+) to (-)

rel min @ (3, -27) b/c y' chgs from (-) to (+)

p.o.i. @ (3/2, -27/2) b/c y'' chgs sign



3) $dV/dt = 72\pi \text{ in}^3/\text{sec}$

What have we learned??

- Can I evaluate limits as x approaches ∞ or -∞ for a variety of functions?