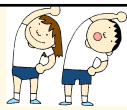
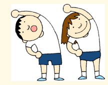


WARM UP!!



GIVEN: $f(x) = \dfrac{(x^2 + 1)}{x}$, find:

a) all critical values

b) all relative extrema

c) extrema on the interval [1/2, 3]

d) all values of c that verify the Mean Value Theorem on the interval [1/2, 3]

a) ±1 (0 is not in the domain)

b) rel max at (-1, -2) b/c f '(x) chgs from (+) to (-)

rel min at (1, 2) b/c f '(x) chgs from (-) to (+)

c) max = 10/3, min = 2

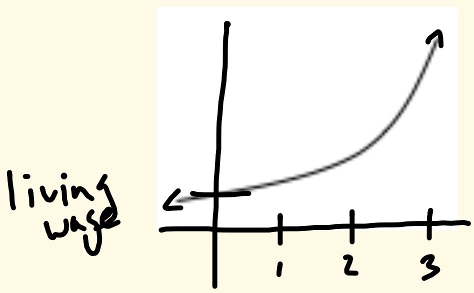
d) $c = \sqrt{3/2}$

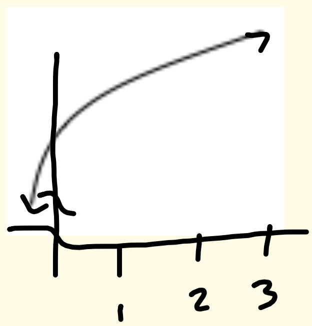
3.4a Concavity!!

ESSENTIAL LEARNING TARGETS

At the end of this lesson you will be able to:

- use the second derivative to determine intervals of concavity and points of inflection
- know how f, f ' and f '' are related to one another





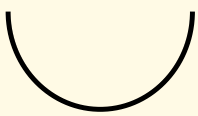
How are these functions similar?

How are they different?

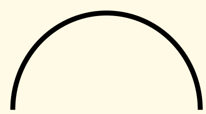
Which one would you want to represent your salary over time? Why?

A little vocabulary:

- a function is concave up if its slopes are increasing
- a function is concave down if its slopes are decreasing

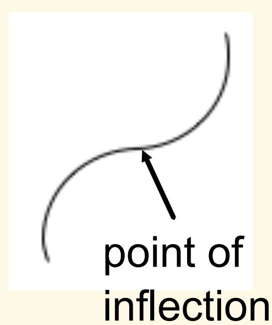


up like a cup



down like a frown

- the point at which a function changes concavity is called a point of inflection



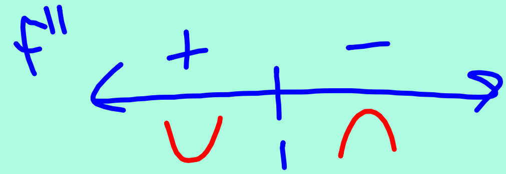
Hmmm, does every change in concavity result in a point of inflection? Why or why not?

In your groups, determine the intervals on which $f(x) = -x^3 + 3x^2 - 2$ is concave up and/or concave down. Then state the point(s) of inflection.

$f'(x) = -3x^2 + 6x$

$f''(x) = -6x + 6 = 0$

$x = 1$



$f(x)$ is ∪ on $(-\infty, 1)$ b/c $f''(x) > 0$

$f(x)$ is ∩ on $(1, \infty)$ b/c $f''(x) < 0$

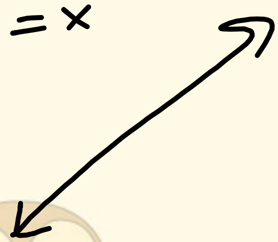
p.o.i. occur @ $(1, 0)$ b/c $f''(x)$ chgs sign

↑ orig funct

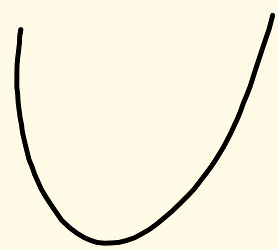
Hmmm: Does a point of inflection occur every time f ''(x) = 0? Can you think of a function where f ''(x) = 0 but there is not a point of inflection?

(Hint, sometimes simplest is best) :)

$y = x$



$y = x^4$



<u>Concavity Test</u>

1) Find f '(x) and f ''(x)

2) Make a sign line for f ''(x)

3) Write answers with because statements

- f(x) is concave up on (a, b) b/c $\underline{\quad f''(x) > 0 \quad}$ on (a, b)
- f(x) is concave down on (a, b) b/c $\underline{\;f''(x) < 0\;}$ on (a, b)
- a p.o.i. occurs at (x, y) b/c $\underline{f''(x) \text{ changes sign}}$ at that point

So,

If f(x) is concave up <=> f '(x) is increasing <=> f ''(x) > 0

If f(x) is concave down <=> f '(x) is decreasing <=> f ''(x) < 0

Can we put this in chart form?

f	↗	↘	∪	∩
f'	+	−	↗	↘
f''			+	−

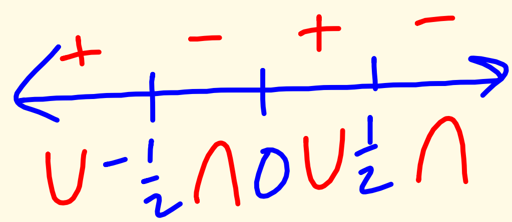
ex) State all intervals of concavity and find all points of inflection for $f(x) = -2x^5 + (5/3)x^3$.

$f'(x) = -10x^4 + 5x^2$

$f''(x) = -40x^3 + 10x = 0$

$-10x(4x^2 - 1) = 0$

$x = 0, \pm\frac{1}{2}$



f is ∪ on $(-\infty, -\frac{1}{2}) \cup (0, \frac{1}{2})$ b/c $f'' > 0$

f is ∩ on $(\frac{1}{2}, 0) \cup (\frac{1}{2}, \infty)$ b/c $f'' < 0$

poi occur @ $x = -\frac{1}{2}, 0, \frac{1}{2}$ b/c f'' chgs sign

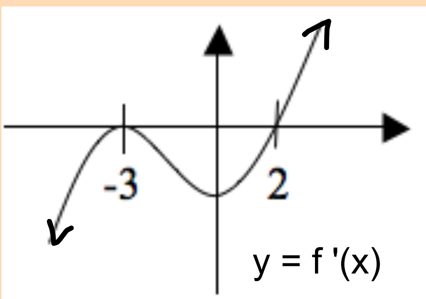
Take it up a notch! The <u>first derivative</u> is graphed below.
Use the graph to determine:

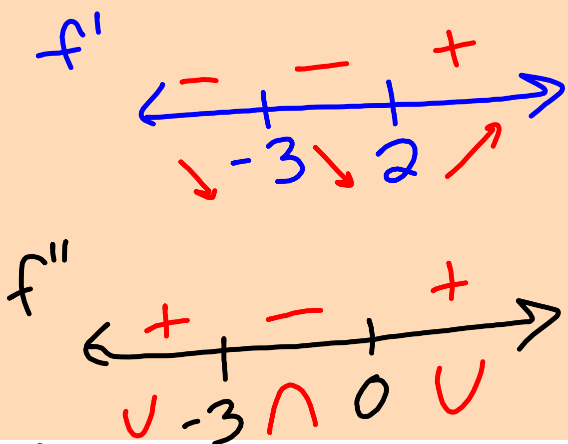
a) intervals of increase/decrease

b) x-values of relative extrema

c) intervals of concavity

d) x-values of points of inflection





f is dec on $(-\infty,-3)\cup(-3,2)$ b/c $f'<0$

f is inc on $(2,\infty)$ b/c $f'>0$

no rel max

rel min occurs @ $x=2$ b/c f' chgs from (−) to (+)

f is ∪ on $(-\infty,-3)\cup(0,\infty)$ b/c $f''>0$

f is ∩ on $(-3,0)$ b/c $f''<0$

poi occur @ $x=-3, 0$ b/c f'' chgs sign

You try! State all intervals of concavity and find all points of inflection for $f(x) = x\sqrt{x+1}$.

$$f'(x) = \sqrt{x+1} + x\left(\tfrac{1}{2}\right)(x+1)^{-\frac{1}{2}}(1)$$

$$= \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$$

$$= \frac{2x+2+x}{2\sqrt{x+1}} = \frac{3x+2}{2\sqrt{x+1}}$$

$$f''(x) = \frac{3\left(2\sqrt{x+1}\right) - (3x+2)(2)\left(\tfrac{1}{2}\right)(x+1)^{-\frac{1}{2}}(1)}{4(x+1)}$$

$$= \frac{6\sqrt{x+1} - \frac{3x+2}{\sqrt{x+1}}}{4(x+1)} \cdot \frac{\sqrt{x+1}}{\sqrt{x+1}} = \frac{6(x+1) - (3x+2)}{4(x+1)\sqrt{x+1}}$$

$$= \frac{3x+4}{4(x+1)\sqrt{x+1}}$$

$f'' = 0$ @ $x = -\frac{4}{3}, -1$

f'': number line starting at -1, $+$ to the right, U

f is U on $(-1, \infty)$ b/c $f'' > 0$

no poi.

Should we try one more? Yes! State all intervals of concavity and find all points of inflection for $f(x) = \sin x + \cos x$ on $[0, 2\pi]$.

How about a little review? :)

1. Find d/dx $[\sec^4(\cos(3x^2))]$

2. $f(x) = \begin{cases} ax^2 - bx + 4, & 3 \le x < \infty \\ 2ax + 5, & -\infty < x < 3 \end{cases}$ and is differentiable for all x. Find a & b.

3. The surface area of a cube is increasing at a rate of 5 ft^2 / sec. How fast is the volume increasing when the surface area is 200 ft^2?

What have we learned?

- What are the steps for the concavity test?
- If $f''(x) < 0$, then $f(x)$ is ?
- If $f''(x)$ changes from negative to positive at (2, 3), then $f(x)$ has a ____________ at (2, 3).