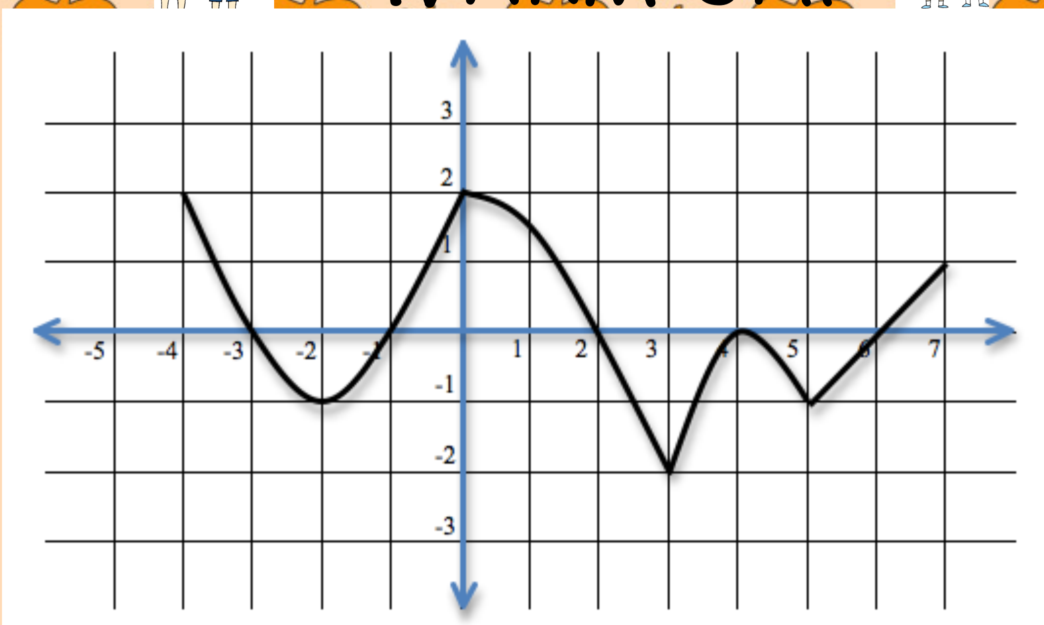
WARM UP!!



1. Is f(x) continuous over [-4, 7]?
2. Is f(x) differentiable over (-4, 7)?
3. List all x-values for which f(x) = 0.
4. List all critical values
5. List all relative extrema
6. List all absolute extrema

1. yes
2. no
3. x = -3, -1, 2, 4, 6
4. x = -2, 0, 3, 4, 5
5. (-2, -1), (0, 2), (3, -2), (4, 0), (5, -1)
6. max = 2, min = -2

3.3 The First Derivative Test for Relative Extrema

ESSENTIAL LEARNING TARGETS

At the end of this lesson you will be able to:

- use the first derivative to determine intervals of increase or decrease
- Use the first derivative to determine local (relative) extrema
- know how f and f' are related to one another
- identify key features of functions and their derivatives related to their graphical, numerical and analytical representations

2nd warmup! NO CALCULATORS!

The Ghoulish Getup Costume Company has sales that can be modeled by $S(x) = 2x^3 - 21x^2 + 60x + 10$, where x = the number of years since 2005 and S (x) is sales in thousands of dollars.

a) Determine the interval(s) of years where sales were increasing.

b) Determine the interval(s) of years where sales were decreasing.

c) When did sales reach a maximum?

d) When did sales reach a minimum?

e) How does the future of the company look?

a) $S'(x) = 6x^2 - 42x + 60 = 0$

$x^2 - 7x + 10 = 0$

$(x-2)(x-5) = 0$

$x = 2, 5$

$S'(x)$: + on (0, 2), − on (2, 5), + after 5; increasing, decreasing, increasing

Sales are inc from 2005 to 2007 and 2010 to ∞

b) Sales are dec from 2007 to 2010

c) max @ 2007

d) min @ 2010

e) pretty damn good

Quick review:

If f(x) is increasing $\Leftrightarrow$ f '(x) > 0

If f(x) is decreasing $\Leftrightarrow$ f '(x) < 0

ALL relative extrema must be located at a:
critical value

First Derivative Test for Relative Extrema

1) Find all critical numbers

2) Make a first derivative sign line

3) Write answers <u>with because statements</u>

   a) f(x) is inc. on (a, b) b/c f '(x) > 0

   b) f(x) is dec. on (a, b) b/c f '(x) < 0

   c) a rel. max. occurs at (x, y) b/c f '(x) chgs from (+) to (-)

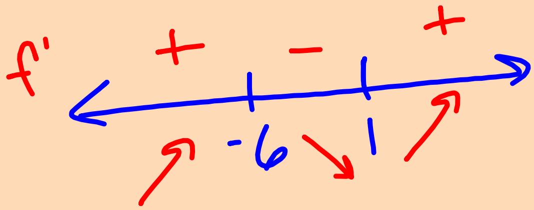
   d) a rel. min. occurs at (x, y) b/c f '(x) chgs from (-) to (+)

ex) Determine the intervals on which $f(x) = {}^{1}/_{3}\, x^3 + {}^{5}/_{2}\, x^2 - 6x - 3$ is increasing and/or decreasing. Then locate all relative extrema.

$f'(x) = x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x = -6, 1$



f(x) is inc on $(-\infty, -6) \cup (1, \infty)$ b/c $f'(x) > 0$

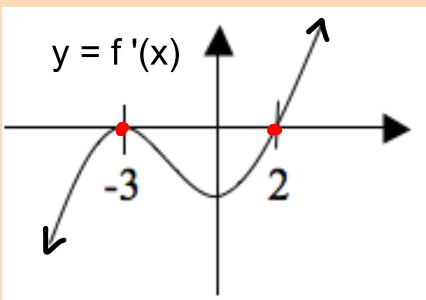
f(x) is dec on $(-6, 1)$ b/c $f'(x) < 0$

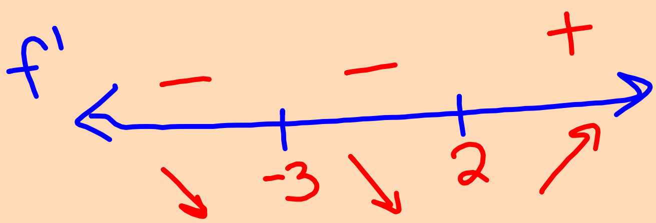
rel max occurs @ $(-6, 51)$ b/c $f'(x)$ chgs from + to −

rel min occurs @ $(1, -\frac{37}{6})$ b/c $f'(x)$ chgs from − to +

You try! Determine the intervals on which $f(x) = (5x + 2) / (x - 3)$ is increasing and/or decreasing. Then locate all relative extrema.

Take it up a notch! The <u>first derivative</u> is graphed below. Use it to state the intervals on which f(x) is increasing and/or decreasing. Then state the x-values of all relative extrema.





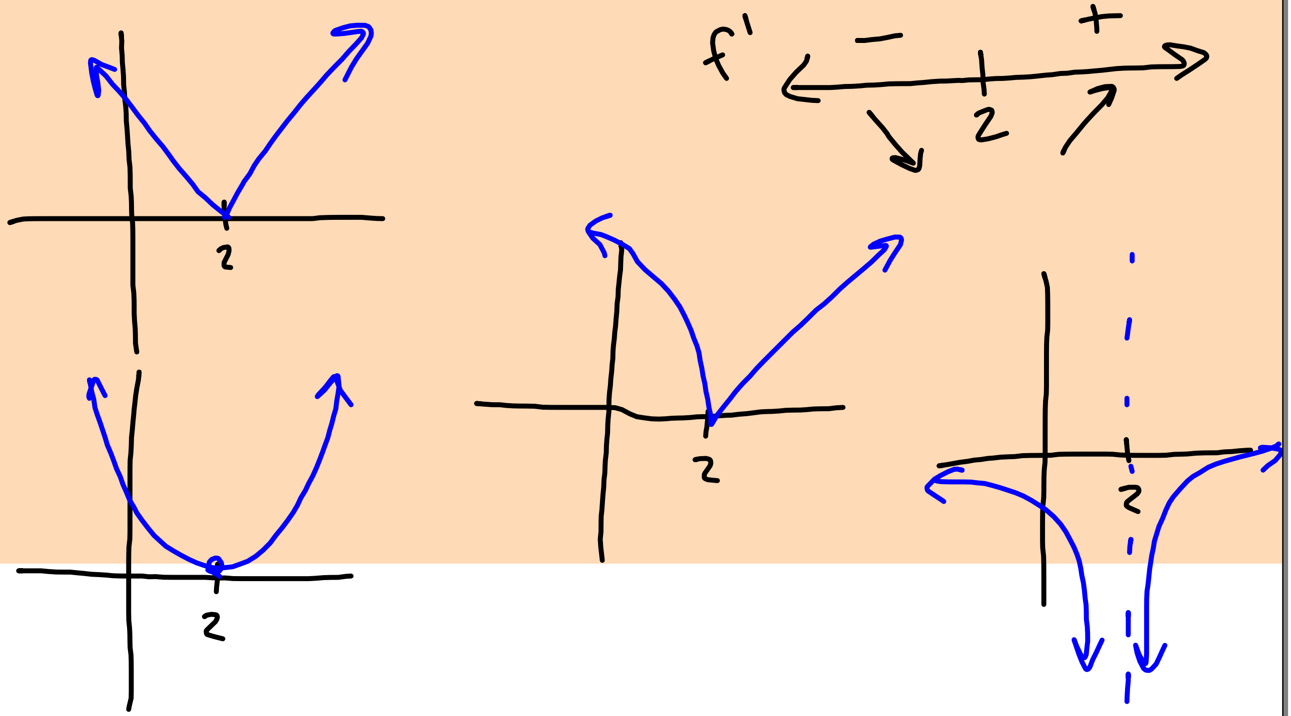
f(x) is inc on $(2,\infty)$ b/c $f'(x) > 0$

f(x) is dec on $(-\infty,-3)\cup(-3,2)$ b/c $f'(x) < 0$

a rel min @ $x=2$ b/c f'(x) chgs from − to +

no rel max b/c f'(x) never chgs from + to −

One more notch! Sketch a graph of f(x) for which f '(x) < 0 on (-∞, 2), f '(x) > 0 on (2, ∞), and f '(x) is undefined at x = 2.



What have we learned?

- What are the steps for the first derivative test?
- If $f'(x) < 0$, then $f(x)$ is ?
- If $f'(x)$ changes from negative to positive at (2, 3), then $f(x)$ has a ____________ at (2, 3).

October 06, 2016