WARMUP: If I drive to school and my average velocity is 60 mph, does this imply that, at least one moment, my actual (instantaneous) velocity was 60 mph somewhere during the trip? Why or why not?

3.2 Mean Value Theorem (and Rolle's Theorem)

ESSENTIAL LEARNING TARGETS

At the end of this lesson you will be able to:

- Apply the Mean Value Theorem to describe the behavior of a function over an interval.
- Show that the hypotheses of the MVT have been satisfied before stating the conclusion.

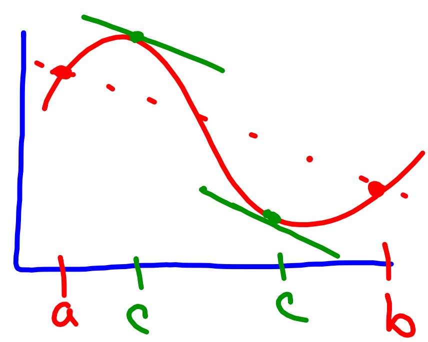
Mean Value Theorem (MVT): If f(x) is continuous on [a, b] and differentiable on (a, b), then there exists at least one value, c in (a, b), such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Hypothesis: f(x) is continuous on [a, b] ✓

f(x) is differentiable on (a, b) ✓

Conclusion: $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Visual Representation:



In other words, there must be at least one place <u>inside</u> of the interval (not at an endpoint) where the slope of the tangent (derivative) = the slope of the secant line connecting the endpoints

(In other other words, where the instantaneous rate of change = the average rate of change) ex) Determine if the Mean Value Theorem can be applied to $f(x) = 2x^3 + x + 4$ on the interval [-2, 1]. If so, find the value of c based on the theorem.

f(x) is cont on [-2, 1] ✓

$f'(x) = 6x^2 + 1$

f(x) is diff on (-2, 1) ✓

$S.O.S = \frac{f(1) - f(-2)}{1 - -2} = 7$

$6x^2 + 1 = 7$

$x^2 = 1$

$x = \pm 1$

$c = -1$

You try! A swimming pool is being filled with a hose. A differentiable function, w(t) represents the number of gallons of water in the pool at time t. Various values of w(t) are indicated in the table below.

time (minutes	0	60	120	180	240
w(t) water (gallons)	0	140	320	530	720

a) Explain why there must be at least one time in (0, 240) when the pool was filling at a rate of 3 gallons per minute.

b) Explain why there must be at least one time in (60, 120) when there were 200 gallons of water in the pool.

a) w(t) is diff on (0, 240) and ∴

w(t) is cont on [0, 240]

$$\frac{w(240)-w(0)}{240-0} = \frac{720}{240} = 3$$

∴ by the MVT there must be at least one time in (0, 240) when the pool was filling @ 3 gal/min

b) w(t) is cont on [60, 120] ✓

w(60) = 140

w(120) = 320

$w(60) \leq 200 \leq w(120)$ ✓

∴ by the IVT there must be at least one time in [60, 120] when there were 200 gals of water in the pool

Rolle's Theorem: If f(x) is continuous on [a, b], differentiable on (a, b), and f(a) = f(b), then there exists at least one point, c in (a, b), such that f '(c) = 0.

Break it down

Conditions (if): f(x) is continuous on [a, b] ✓

f(x) is differentiable on (a, b) ✓

f(a) = f(b) ✓

Result (then): f '(c) = 0

Ex) Determine if Rolle's theorem can be applied to $f(x) = x^2 - 3x$ on the interval [1, 2]. If so, find the value of c such that $f'(c) = 0$.

$f(x)$ is cont on $[1, 2]$ ✓

$f'(x) = 2x - 3$

$f(x)$ is diff on $(1, 2)$ ✓

$f(1) = -2$

$f(2) = -2$

$f(1) = f(2)$ ✓

∴ Rolle's applies

$2x - 3 = 0$

$x = \frac{3}{2}$

$c = \frac{3}{2}$

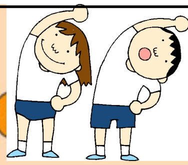
You try! Suppose f(x) is a differentiable function with selected values shown below. Explain why f(x) must have at least 2 horizontal tangents.

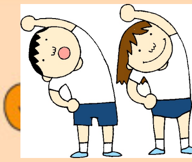
x	1	3	6	8	9	12	15	20
f(x)	-3	5	8	5	2	-1	2	6

f(x) is diff and ∴ cont

b/c $f(3) = f(8) = 5$ and $f(9) = f(15) = 2$ so slope of secant is 0 from 3 to 8 and from 9 to 15

∴ by the MVT f(x) must have @ least 2 horiz. tangents





REVIEW!!

If Line A is tangent to $3xy + y^2 = -5$ at (-2, 1) and Line B is normal to $4x^2y - 2y^2x = -6$ at (1, 3), find the x-value of the point of intersection of these two lines.

✓ $x = 22/25$

What have we learned?

- What are the 2 conditions for MVT to apply?
- What is the conclusion of the MVT?
- What are the 3 conditions for Rolle's to apply?
- What is the conclusion of Rolle's Theorem?