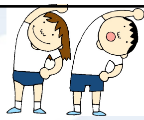
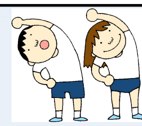


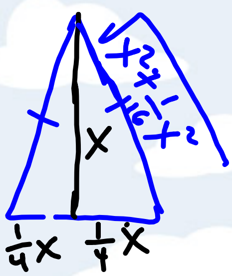
WARM UP!!



An isosceles triangle's base (and shortest side) is half its height. If the height is growing at a rate of 4 in/sec:

a) How fast is the triangle's area growing when the height is 10 inches?

b) How fast is the triangle's perimeter growing when the height is 10 inches?



a) $A = \frac{1}{4}x(x) = \frac{1}{4}x^2$

$\frac{dA}{dt} = \frac{1}{2}x\frac{dx}{dt} = \frac{1}{2}(10)(4) = 20\ \frac{in^2}{sec}$

$\frac{dx}{dt} = 4$

$x = 10$

$\frac{dA}{dt} = ?$

b) $P = \frac{1}{2}x + 2\sqrt{x^2 + \frac{x^2}{16}}$

$P = \frac{1}{2}x + 2\sqrt{\frac{17x^2}{16}} = \frac{1}{2}x + \frac{1}{2}x\sqrt{17}$

$\frac{dP}{dt} = \frac{1}{2}\frac{dx}{dt} + \frac{1}{2}\sqrt{17}\frac{dx}{dt} = 2 + 2\sqrt{17}$

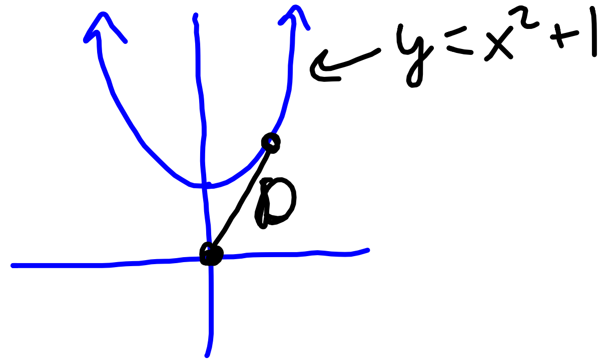
✓

a) 20 in^2 / sec

b) $2 + 2\sqrt{17}$ in / sec

HW 13

$$\frac{dx}{dt} = 2$$



$$\frac{dD}{dt} = ?$$

$$D = \sqrt{x^2 + y^2}$$

$$D = \boxed{\sqrt{x^2 + (x^2+1)^2}}$$

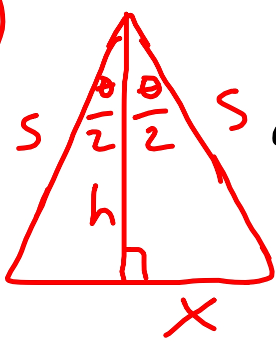
$$D^2 = x^2 + (x^2+1)^2$$

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2(x^2+1)\left(2x\frac{dx}{dt}\right)$$

$$2D\frac{dD}{dt} = 4x + 2(x^2+1)(4x)$$

$$\frac{dD}{dt} = \frac{4x + 8x(x^2+1)}{2D} = \frac{2x + 4x(x^2+1)}{D}$$

17



a) $x = s\sin\frac{\theta}{2}$

$h = s\cos\frac{\theta}{2}$

$A = x \cdot s \cdot \cos\frac{\theta}{2}$

$A = s \cdot s \cdot \sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$A = s^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$A = \frac{1}{2}s^2\sin\theta$

constant

b) $\frac{d\theta}{dt} = \frac{1}{2}$

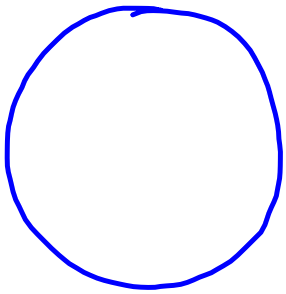
$\frac{dA}{dt} = \frac{1}{2}s^2\cos\theta \frac{d\theta}{dt}$

$= \frac{1}{2}s^2\cos\frac{\pi}{6}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{8}s^2$

c) rate of change of area depends on size of θ at each moment

19



$$\frac{dV}{dt} = 800$$

$$\frac{dr}{dt} = ?$$

$$r = 30$$

$$V = \frac{4}{3}\pi r^3$$

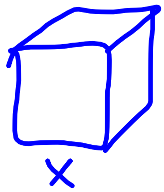
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$800 = 4\pi(30)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{8\cancel{00}}{4\pi\, 9\cancel{00}} = \frac{8}{36\pi}\,\frac{cm}{min}$$

$$= \frac{2}{9\pi}\,\frac{cm}{min}$$

21



$$\frac{dS}{dt} = ?$$

$$x = 1$$

$$\frac{dx}{dt} = 3$$

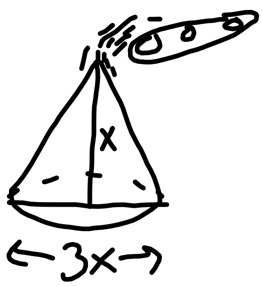
$$S = 6x^2$$

$$\frac{dS}{dt} = 12x\frac{dx}{dt}$$

$$= 12(1)(3)$$

$$= 36 \ \frac{cm^2}{sec}$$

23



$$\frac{dV}{dt} = 10$$

$$\frac{dx}{dt} = ?$$

$$x = 15$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{3}{2}x\right)^2(x)$$

$$V = \frac{3}{4}\pi x^3$$

$$\frac{dV}{dt} = \frac{9}{4}\pi x^2\frac{dx}{dt}$$

$$10 = \frac{9}{4}\pi(15)^2\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{\overset{2}{\cancel{10}}\cdot 4}{9\pi\underset{3}{\cancel{15}}\cdot 15} = \frac{8}{405\pi}\ \frac{ft}{min}$$

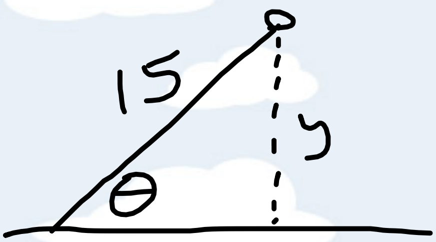
2.6b Related Rates Continued

At the end of this lesson you will be able to:

- Set up and solve related rates problems that require a substitution of variables

2nd warmup: A pole, with a ball at the top, is being lifted off the ground to a vertical position. The angle the base of the pole makes with the ground is changing at a rate of 3 degrees per second. If the pole is 15 feet long, how fast is the distance from the ball to the ground changing at the time the angle is 36 degrees?

(Note: ALL calculus problems MUST be done in radians so you will need to convert.)



$$\frac{d\theta}{dt} = 3^\circ/\text{sec} = \frac{\pi}{60}\,\frac{\text{rad}}{\text{sec}}$$

$$\frac{dy}{dt} = ?$$

$$\theta = 36^\circ = \frac{\pi}{5}\text{ rad}$$

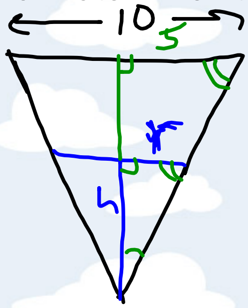
$$\sin\theta = \frac{y}{15} = \frac{1}{15}y$$

$$\cos\theta\,\frac{d\theta}{dt} = \frac{1}{15}\,\frac{dy}{dt}$$

$$\cos\frac{\pi}{5}\left(\frac{\pi}{60}\right) = \frac{1}{15}\,\frac{dy}{dt}$$

$$\frac{dy}{dt} = \cancel{15}\left(\cos\frac{\pi}{5}\right)\left(\frac{\pi}{\cancel{60}_{4}}\right)\ \frac{\text{ft}}{\text{sec}}$$

ex) A conical tank (vertex down) is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of water when the water is 8 feet deep.



$$\frac{5}{12} = \frac{r}{h}$$

$$r = \frac{5}{12}h$$

$\frac{dV}{dt} = 10$

$\frac{dh}{dt} = ?$

$h = 8$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 h$$

$$V = \frac{25}{3 \cdot 144}\pi h^3$$

$$\frac{dV}{dt} = \frac{25}{144}\pi h^2 \frac{dh}{dt}$$

$$10 = \frac{25}{144}\pi(64)\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{\overset{2}{\cancel{10}} \cdot \overset{9}{\cancel{144}}}{\underset{5}{\cancel{25}} \cdot \pi \cdot \underset{\underset{2}{\cancel{4}}}{\cancel{64}}} = \frac{9}{10\pi}\ \frac{\text{ft}}{\text{min}}$$

OR

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{2}{3}\pi r \frac{dr}{dt} h + \frac{1}{3}\pi r^2 \frac{dh}{dt}$$

$$r = \frac{5}{12}h$$

$$\frac{dr}{dt} = \frac{5}{12}\frac{dh}{dt}$$

?

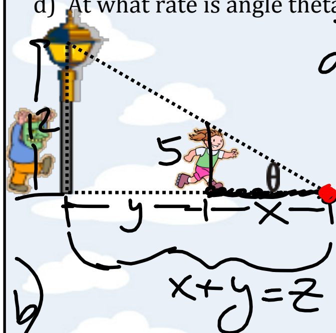
Michael Myers is chasing his sister, Laurie, through the streets of Chicago on Halloween night. Laurie, who is 5 feet tall, is running at a rate of 14 feet per second when she passes a light post that is 12 feet tall.

a) At what rate is the length of Laurie's shadow changing?

b) At what rate is the tip of Laurie's shadow moving?

c) A triangle is formed by the light post and the tip of Laurie's shadow. At what rate is the area of this triangle changing?

d) At what rate is angle theta changing when theta is 30 degrees?



a) $\frac{dx}{dt} = ?$ $\qquad \frac{5}{12} = \frac{x}{x+y}$

$\frac{dy}{dt} = 14$

$5x + 5y = 12x$

$5y = 7x$

$5\frac{dy}{dt} = 7\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{5(14)}{7} = 10 \frac{ft}{sec}$

b) $\frac{dz}{dt} = ?$

$x + y = z$

$\frac{dx}{dt} + \frac{dy}{dt} = \frac{dz}{dt}$

$\frac{dz}{dt} = 10 + 14 = 24 \frac{ft}{sec}$

c) $A = \frac{1}{2} z(12) = 6z$

$\frac{dA}{dt} = 6\frac{dz}{dt} = 6(24) = 144 \frac{ft^2}{sec}$

d) $\frac{d\theta}{dt} = ?$ $\qquad \cot\theta = \frac{x}{5}$

$\theta = \frac{\pi}{6}$

$-\csc^2\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$

$-4\frac{d\theta}{dt} = \frac{1}{5}(10)$

$\frac{d\theta}{dt} = -\frac{1}{2} \frac{rad}{sec}$

What have we learned?

- Can I set up any and all related rates problems correctly and use appropriate steps to solve them?