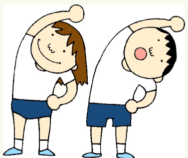
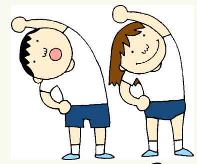


WARM UP!!



Find the exact value of the point of intersection of the lines tangent to $y = \cos x$ at $\pi/2$ and $y = \sin x$ at 2π

$\left(\frac{\pi}{2}, 0\right)$

$y' = -\sin x$

$y'\left(\frac{\pi}{2}\right) = -\sin\frac{\pi}{2}$

$= -1$

$y = -1\left(x - \frac{\pi}{2}\right)$

$y = -x + \frac{\pi}{2}$

$(2\pi, 0)$

$y' = \cos x$

$y'(2\pi) = \cos 2\pi$

$= 1$

$y = x - 2\pi$

$-x + \frac{\pi}{2} = x - 2\pi$

$2x = 2\pi + \frac{\pi}{2}$

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$y = \frac{5\pi}{4} - 2\pi = \frac{-3\pi}{4}$

$\boxed{\left(\frac{5\pi}{4}, \frac{-3\pi}{4}\right)}$

HW (63) $f(x) = x^2 - Kx$ $\quad y = 4x - 9$

equate the slopes → $f'(x) = 2x - K = 4 \rightarrow K = 2x - 4$

equate the y-values → $x^2 - Kx = 4x - 9$

$x^2 - (2x-4)(x) = 4x - 9$

$x^2 - 2x^2 + 4x = 4x - 9$

$x^2 - 9 = 0$ $\quad \boxed{K = 2, -10}$

$x = \pm 3$

65 $f(x) = \frac{K}{x} = Kx^{-1}$

$y = -\frac{3}{4}x + 3$

equate the y's → $\frac{K}{x} = -\frac{3}{4}x + 3$

equate the slopes → $\frac{-K}{x^2} = \frac{-3}{4}$

$\cancel{-}4K = \cancel{-}3x^2$

$K = \frac{3}{4}x^2$

$\frac{3x^2}{4x} = -\frac{3}{4}x + 3$

$\frac{3}{4}x = -\frac{3}{4}x + 3$

$\frac{3}{2}x = 3$

$x = 2$

$K = \frac{3}{4}(4) = 3$

77 $f(x)=\sqrt{x}$ $(x_0,y_0)=(-4,0)$

$$f'(x)=\frac{0-y}{-4-x}=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}}=\frac{y}{4+x}$$

$$y=\frac{4+x}{2\sqrt{x}}=\sqrt{x}$$ equate the y's

$4+x=2x$ $f'(4)=\frac{1}{4}$

$x=4$

pt $(4,2)$ $y-2=\frac{1}{4}(x-4)$

93 $s(t) = -16t^2 + v_0 t + s_0$

$t=0$

$s=0$

a) $s(t) = -16t^2 + 1362$

$v(t) = s'(t) = -32t$

b) $\text{avg vel} = \dfrac{s(2) - s(1)}{2-1} = \dfrac{1298 - 1346}{1}$

$= -48 \frac{ft}{sec}$

c) $v(1) = -32 \frac{ft}{sec}$ $v(2) = -64 \frac{ft}{sec}$

d) $-16t^2 + 1362 = 0$

$t \approx 9.226$ sec

e) $v(9.226)$

$\approx -295.242 \frac{ft}{sec}$

109 $y=ax^2+bx+c$

(0,1)
tangent to
$y=x-1$
at
(1,0)

pt (0,1) $1=0+0+c$

$y=ax^2+bx+1$

pt (1,0) $0=a+b+1$

$y'=2ax+b$ slope of tangent

$y'(1)=2a+b=1$

$a+b=-1$

$a=2$

$2+b=-1$

$b=-3$

$\boxed{y=2x^2-3x+1}$

111 $y = x^3 - 9x$ $(1,-9)$

$y' = 3x^2 - 9$

$y + 9 = (3x^2 - 9)(x-1)$

$y = (3x^2-9)(x-1) - 9 = x^3 - 9x$

$3x^3 - 3x^2 - 9x + 9 - 9 = x^3 - 9x$

$2x^3 - 3x^2 = 0$ $y'(0) = -9$

$x^2(2x-3) = 0$ $y'\left(\frac{3}{2}\right) = -\frac{9}{4}$

$x = 0, \frac{3}{2}$

$y + 9 = -9(x-1)$

$y + 9 = -\frac{9}{4}(x-1)$

113

$$f(x)=\begin{cases} ax^3, & x\le 2 \\ x^2+b, & x>2 \end{cases}$$

Continuity

$8a = 4+b$

$\frac{8}{3} = 4+b$

$\boxed{b = \frac{-4}{3}}$

differentiability

$$f'(x)=\begin{cases} 3ax^2, & x<2 \\ 2x, & x>2 \end{cases}$$

$12a = 4$

$\boxed{a = \frac{1}{3}}$

2.3a Product and Quotient Rules!!

At the end of this lesson, you will be able to:

- Recognize when product and quotient rules are necessary to use
- Evaluate derivatives that require product and quotient rules

Suppose f(x) = x^3 and g(x) = x^2.

What would the derivative of f(x) * g(x) be?

$$\cancel{f \cdot g = x^3 \cdot x^2}$$
$$\cancel{(f \cdot g)' = 3x^2 \cdot 2x}$$
$$\cancel{= 6x^3}$$

$$f \cdot g = x^3 \cdot x^2$$
$$= x^5$$
$$(f \cdot g)' = \underline{5x^4}$$

Product Rule: $[f(x)*g(x)]' = f'(x)\,g(x) + f(x)\,g'(x)$

ex) $y = \overbrace{(3x - 2x^2)}^{f}\overbrace{(5 + 4x)}^{g}$, find dy/dx

function whose output is the slope of the line tangent to f(x) at any given point

$$\frac{dy}{dx} = (3-4x)(5+4x) + (3x-2x^2)(4)$$

$$= 15 + 12x - 20x - 16x^2 + 12x - 8x^2$$

$$= -24x^2 + 4x + 15$$

You Try!!

ex) Suppose $f(x) = (8x^3 + 4x)(\sin x)$, find $f\,'(x)$

$$f'(x) = (24x^2 + 4)(\sin x) + (8x^3 + 4x)(\cos x)$$

a) -5 b) infinity c) -5 d) DNE e) 1 f) 1 g) 4

h) 1 i) -2 j) -2 k) und l) -2

Quotient Rule!

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

ex) Find $\frac{d}{dx}\frac{3x-2x^2}{5+4x} = \frac{(3-4x)(5+4x)-(3x-2x^2)(4)}{(5+4x)^2}$

You try!

ex) If $y = \dfrac{\cos x}{8x-6}$, find y'

$$y' = \frac{-\sin x(8x-6) - \cos x(8)}{(8x-6)^2}$$

Let's take it up a notch!!

$$y = \frac{\sin x \cos x}{3x^2 - 5}$$

Find dy/dx.

$$\frac{dy}{dx} = \frac{[\cos x \cdot \cos x + \sin x(-\sin x)](3x^2-5) - \sin x \cos x(6x)}{(3x^2-5)^2}$$

$$= \frac{\cos(2x)(3x^2-5) - 3x\sin(2x)}{(3x^2-5)^2}$$

Can you rewrite a function?

Differentiate the following WITHOUT using product or quotient rules.

$$y = \frac{6}{5x^3} = \frac{6}{5}x^{-3} \qquad y' = -\frac{18}{5}x^{-4} = \frac{-18}{5x^4}$$

$$y = \frac{6x^3 - 8x + 7}{5} = \frac{1}{5}(6x^3 - 8x + 7)$$

$$y' = \frac{1}{5}(18x^2 - 8)$$

What have we learned??

- What are the product and quotient rules? Can I say them from memory?
- When do I need to use the product and quotient rules? When do I NOT need to use them?

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