Answer Key - 2003

5	31-45
4	23-30
3	16-22
2	8-15
1	0-7

A.P. Calculus Final Exam Answer Sheet – no calculator portion

1. E
2. D
3. E
4. D
5. D
6. C
7. B
8. B
9. A
10. B
11. C
12. E
13. A
14. E
15. D
16. C
17. A
18. A
19. D
20. D
21. A
22. D
23. E
24. C
25. E
26. B
27. B
28. E

Answer Key - 2003

A.P. Calculus Final Exam Answer Sheet – calculator portion

76. C
77. C
78. C
79. D
80. B
81. D
82. A
83. A
84. A
85. A
86. B
87. B
88. C
89. D
90. B
91. E
92. D

2003 Multiple Choice No Calculator Answer Key

1) $y = (x^3+1)^2$ so $\frac{dy}{dx} = 2(x^3+1)^1(3x^2) = 6x^2(x^3+1)$ $\boxed{E}$

2) $\int_0^1 e^{-4x}dx = -\frac{1}{4}\int_0^{-4} e^u du = \frac{1}{4}\int_{-4}^{0} e^u du = \frac{1}{4}e^u\Big]_{-4}^{0} = \frac{1}{4}e^0 - \frac{1}{4}e^{-4}$

$u = -4x$
$du = -4dx$

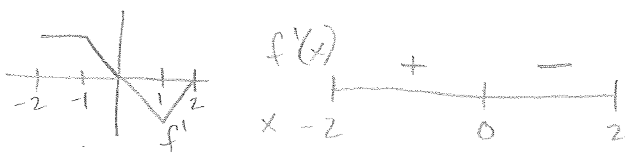
$= \frac{1}{4} - \frac{e^{-4}}{4}$ $\boxed{D}$

3) $\lim_{x\to\infty} f(x) = 2$ implies a H.A. at $y=2$ $\boxed{E}$

4) $y = \frac{2x+3}{3x+2}$ $\frac{dy}{dx} = \frac{2(3x+2)-(2x+3)(3)}{(3x+2)^2} = \frac{6x+4-6x-9}{(3x+2)^2} = \frac{-5}{(3x+2)^2}$ $\boxed{D}$

5) $\int_0^{\pi/4} \sin x\,dx = -\cos x\Big]_0^{\pi/4} = -\cos\frac{\pi}{4} - -\cos 0 = -\frac{\sqrt{2}}{2}+1$ $\boxed{D}$

6) $\lim_{x\to\infty} \frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1} = \frac{1}{4}$ $\boxed{C}$

7) 

so $f(x)$ is inc. on $(-2,0)$ $\boxed{B}$

8) $\int x^2\cos(x^3)dx = \frac{1}{3}\int \cos u\,du = \frac{1}{3}\sin u + C = \frac{1}{3}\sin(x^3)+C$

$u = x^3$ $du = 3x^2dx$

$\boxed{B}$

9) $f(x) = \ln(x+4+e^{-3x})$ so $f'(x) = \frac{1}{x+4+e^{-3x}}\cdot(1-3e^{-3x})$

$f'(0) = \frac{1-3e^0}{4+e^0} = \frac{1-3}{4+1} = \frac{-2}{5}$ $\boxed{A}$

10) $f(x)<0 \Rightarrow$ below x-axis
$f'(x)<0 \Rightarrow$ decreasing $f''(x)<0 \Rightarrow$ concave down $\boxed{B}$

11) $u = 2x+1$, $\int_0^2 \sqrt{2x+1}\,dx = \frac{1}{2}\int_1^5 \sqrt{u}\,du$ $\boxed{C}$

$du = 2dx$

12) $\frac{dV}{dt} = K\sqrt{V}$ $\boxed{E}$

rate of change of volume

is prop. to

the square root of the volume

13) $\boxed{A}$ continuous but with a sharp turn

14) $y = x^2 \sin 2x \qquad \frac{dy}{dx} = 2x\sin 2x + x^2\cos 2x \cdot 2 = 2x(\sin 2x + x\cos 2x)$

$\boxed{E}$

15) $f'(x) = x^2 - \frac{2}{x} = \frac{x^3-2}{x}$

$f'(x)$: $+ \quad - \quad +$

x: $0 \quad \sqrt[3]{2}$

$\boxed{D}$

16) slope of tangent $= \frac{7-^-2}{1-^-2} = \frac{9}{3} = 3$ so $f'(1) = 3$ $\boxed{C}$

17) $f(x) = 2xe^x$

$f'(x) = 2e^x + 2xe^x$

$f''(x) = 2e^x + 2e^x + 2xe^x$

$= 4e^x + 2xe^x$

$= 2e^x(2+x) = 0$

$x = ^-2$

$f''(x)$: $- \quad +$

x: $^-2$

$\boxed{A}$

) $g'(x) < 0$ on $(-2, 2)$ only so $\boxed{A}$

19) $\frac{dy}{dx} = 2x + 3$

$\int dy = \int (2x+3)\,dx$

$y = x^2 + 3x + C$

$\Rightarrow 2 = 1 + 3 + C$

$-2 = C$

so $y = x^2 + 3x - 2$

$\boxed{D}$

20) $f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ 4x-7 & \text{if } x > 3 \end{cases} \qquad f'(x) = \begin{cases} 1 & \text{if } x < 3 \\ 4 & \text{if } x > 3 \end{cases} \Rightarrow$ not diff. at $x = 3$

$\lim_{x\to 3^-} f(x) = 5 \quad \lim_{x\to 3^+} f(x) = 5$ so $\lim_{x\to 3} f(x) = 5 = f(3)$ so I + II $\boxed{D}$

21) P.O.I. is where $f''(x)$ changes sign at $x = a, 0$ $\boxed{A}$

22) slope is -6, equation of derivative is $f'(x) = -6x + 6$

$f(x) = \int(-6x+6)\,dx = -3x^2 + 6x + C$

$5 = 0 + 0 + C$ so $f(x) = -3x^2 + 6x + 5$

$f(1) = -3 + 6 + 5 = 8$

$\boxed{D}$

23) $\frac{d}{dx}\int_0^{x^2} \sin(t^3)\,dt = \sin(x^2)^3 \cdot 2x = 2x \sin x^6$ $\boxed{E}$

) $f(x) = 4x^3 - 5x + 3$ $f(-1) = -4 + 5 + 3 = 4$ $y - 4 = 7(x+1)$
$f'(x) = 12x^2 - 5$ $f'(-1) = 12 - 5 = 7$ $y = 7x + 11$ $\boxed{C}$

25) $x(t) = 2t^3 - 21t^2 + 72t - 53$ so at rest at
$v(t) = 6t^2 - 42t + 72 = 0$ $t = 3, 4$
$t^2 - 7t + 12 = 0$
$(t-3)(t-4) = 0$ $\boxed{E}$

26) $3y^2 - 2x^2 = 6 - 2xy$
$6y\frac{dy}{dx} - 4x = -2y - 2x\frac{dy}{dx}$ $\rightarrow 6(2)\frac{dy}{dx} - 4(3) = -2(2) - 2(3)\frac{dy}{dx}$
$12\frac{dy}{dx} - 12 = -4 - 6\frac{dy}{dx}$
$18\frac{dy}{dx} = 8$ $\frac{dy}{dx} = \frac{4}{9}$ $\boxed{B}$

2) $f(x) = x^3 + x$ $(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))}$
given that $2 = x^3 + x$
$x = 1$ $(f^{-1})'(2) = \frac{1}{4}$ $\boxed{B}$
$f'(x) = 3x^2 + 1$
$f'(1) = 3 + 1 = 4$

28) $g'(x) > 0 \Rightarrow$ $g(x)$ is increasing
$g''(x) > 0 \Rightarrow$ concave up $\Rightarrow$ <u>slope</u> <u>is</u> <u>increasing</u>
slope of $(4,12)$ and $(5,18)$ is 6.
So the next point must have a y-value greater than 24. $\boxed{E}$

2003 Multiple Choice Calculator Answer Key

76) $v(t) = 3 + 4.1\cos(0.9t) \rightarrow$ put this in [Y=] and find $\frac{dy}{dx}$ at $x=4 \rightarrow a(4) \approx 1.6329$ $\boxed{C}$

77) $\int_{-3}^{3} (f(x)+1)\,dx = \int_{-3}^{3} f(x)\,dx + \int_{-3}^{3} 1\,dx = (-2+2-2) + x\Big]_{-3}^{3}$ $\boxed{C}$

$= -2 + 3 - {}^{-}3 = 4$

78) $\frac{dr}{dt} = .2 \qquad A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = C \cdot \frac{dr}{dt} = 20\pi(.2) = 4\pi$ $\boxed{C}$

79) $\lim_{x\to 4} f(x)$ exists for I and II because $\lim_{x\to 4^-} f(x) = \lim_{x \to 4^+} f(x)$ $\boxed{D}$

80) cont, diff, but $f(a) \neq f(b)$ so mean value thm. applies but Rolle's does not.

slope of secant $= \frac{4-{}^{-}5}{1-{}^{-}2} = \frac{9}{3} = 3$ so d ✓ $\boxed{B}$

intermediate val. thm. says it must hit all y-values between -5 and 4, so a ✗, c ✓

extreme val. thm. says there must be a max and min so e ✓

81) $f'(x) = \sin(x^2+1)$ graph and see 4 zeroes where it changes sign $\boxed{D}$

82) $r(t) = t^3 - 4t^2 + 6$, $0 \le t \le 8$

think of this as $\frac{dh}{dt}$ or velocity

$r(t)$		+		−		+	
t	0		1.572		3.514		8

$\boxed{A}$

83) $v(t) = e^t + te^t \qquad \frac{1}{3-0}\int_0^3 e^t + te^t\,dt \approx \frac{1}{3}(60.2566)$ $\boxed{A}$

≈ 20.086

84) $\frac{dT}{dt} = -110e^{-.4t}$

$\int dT = \int -110 e^{-.4t}\,dt \quad u = -.4t \quad du = -.4dt$

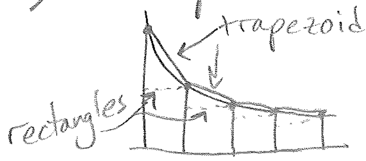
$T = \frac{-110}{-.4} e^{-.4t} + C$

$350 = 275e^0 + C$ so $C = 75$

$T(5) = 275e^{-.4(5)} + 75$ $\boxed{A}$

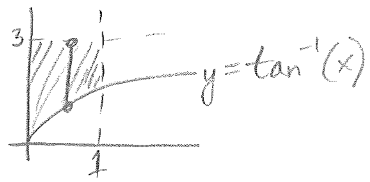
$= 112.217° \approx 112°$

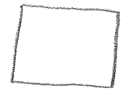
85) trapezoids above, right-handed rectangles below



$\boxed{A}$

86)





side $= 3 - \tan^{-1}(x)$

$V = \int_0^1 (3 - \tan^{-1}(x))^2\,dx$

$= 6.612$ $\boxed{B}$

87) $f'(x) = \frac{\sqrt{x}}{1+x+x^3}$ $\quad f''(x) = \frac{\frac{1}{2\sqrt{x}}(1+x+x^3) - \sqrt{x}(1+3x^2)}{(1+x+x^3)^2}$

$f''(x)$ changes sign at $x \approx .473$ $\boxed{B}$

88) average value $=1$ implies that the height of the rectangle that would produce the same area from 2 to 4 as that under the curve is 1. So the area under the curve is 2. Since C is the only one with an area of exactly 2 $\Rightarrow$ $\boxed{C}$

89) $g(x) = x f(x)$ $\quad$ so $g(2) = 2 \cdot f(2) = 2 \cdot 3 = 6$

$g'(x) = f(x) + x f'(x)$ $\quad$ so $g'(2) = f(2) + 2 \cdot f'(2)$

$= 3 + 2(-5) = -7$ $\boxed{D}$

$y - 6 = -7(x-2)$

90) $f(x)$ is increasing but the slopes are decreasing

$\boxed{B}$

91) $a(t) = \ln(1+2^t) \qquad v(1) = 2$

$$\underbrace{\int_1^2 a(t)\,dt}_{1.346} = v(t)\Big]_1^2 = v(2) - v(1) \qquad \boxed{E}$$

$$= v(2) - 2 \quad \text{so } v(2) = 3.346$$

92) $g(x) = \int_0^x \sin(t^2)\,dt$

$g(x)$ is decreasing when $g'(x)$ is negative

$g'(x) = \sin x^2 \qquad \boxed{D}$

$g'(x)$		+		+	−	+	
x	-1		0	1.772	2.507		3