Warmup

ex) If $y = 3x^2 - 4$, find dy/dt.

$$\frac{dy}{dt} = 6x\frac{dx}{dt}$$

What is dy/dt when x = 2 if dx/dt = -5?

$$\frac{dy}{dt} = 6(2)(-5)$$
$$= -60$$

HW 11 $\sin x + 2\cos(2y) = 1$

$$\cos x - 2\sin(2y)\left(2\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = \frac{\cos x}{4\sin(2y)}$$

13 $\sin x = x(1+\tan y)$

$$\cos x = 1 + \tan y + x(\sec^2 y)\left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{\cos x - 1 - \tan y}{x\sec^2 y}$$

15 $y = \sin(xy)$

$$\frac{dy}{dx} = \cos(xy)\left(y + x\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = y\cos(xy) + x\cos(xy)\frac{dy}{dx}$$

$$\frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy)$$

$$\frac{dy}{dx}\left(1 - x\cos(xy)\right) = y\cos(xy)$$

27 $\tan(x+y)=x \qquad (0,0)$

$$\sec^2(x+y)\left(1+\frac{dy}{dx}\right)=1$$

$$1+\frac{dy}{dx}=\cos^2(x+y)$$

$$\frac{dy}{dx}=\cos^2(x+y)-1$$

$$\text{at }(0,0)=\cos^2(0)-1$$

$$=1-1=0$$

49 $y^2=x^3$

$$2y\frac{dy}{dx}=3x^2$$

$$\frac{dy}{dx}=\frac{3x^2}{2y}$$

$$\frac{d^2y}{dx^2}=\frac{6x(2y)-3x^2\left(2\frac{dy}{dx}\right)}{4y^2}$$

$$=\frac{12xy-6x^2\left(\frac{3x^2}{2y}\right)}{4y^2} \qquad \frac{dy}{dx}$$

$$=\frac{12xy-\frac{\cancel{18}^{9}x^4}{\cancel{2}y}}{4y^2}\cdot\frac{y}{y}$$

$$=\frac{12xy^2-9x^4}{4y^3}$$

2.6 Related Rates

ESSENTIAL LEARNING TARGETS

At the end of this lesson, you will be able to:

- express information about rates of change in applied contexts
- Solve related rates problems

Steps for solving related rates

(there are many methods, but this one is mine)

1) Draw a picture (if applicable) and label dimensions (constants with constants, changing dimensions with variables)

2) Write what you are given

3) Write what you are trying to find

4) Write an equation that relates the variables

5) Check that every variable in your equation is one where you are given the rate or you want the rate (only plug in constants for variables that truly are <u>constant</u>)

6) If you have extraneous variables, find a way to replace them by relating them to other variables and substituting

7) Differentiate with respect to time

8) Substitute and solve

Gotchas

1) Remember that a positive rate indicates that a dimension is getting bigger and a negative rate indicates that a dimension is getting smaller; the sign does not relate to the direction an object is moving.

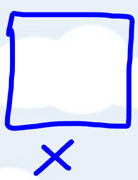
2) Don't forget product rule!!

3) Don't forget your units!!

Reminder

Geometry formulas are all in the ~~front~~ back of the book if you need them. Get familiar with them, because they will not be given to you on any assessments.

ex) The sides of a square are increasing at a rate of 5 cm/min. How fast is the area increasing when the sides measure 15 cm in length?



$\frac{dx}{dt} = 5$

$x = 15$

$\frac{dA}{dt} = ?$

$A = x^2$

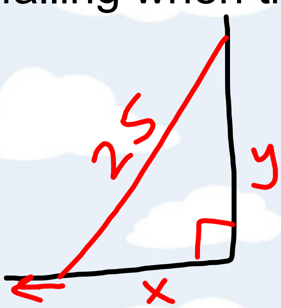
$\frac{dA}{dt} = 2x \frac{dx}{dt}$

$= 2(15)(5)$

$= 150 \ \frac{cm^2}{min}$

You try!

ex) A 25-foot ladder is leaning against a wall. The bottom of the ladder is being pulled horizontally away from the wall at the constant rate of 3 ft/sec. How fast is the top of the ladder falling when the bottom of the ladder is 15 feet from the wall?



$$\frac{dx}{dt} = 3$$

$$x = 15$$

$$\frac{dy}{dt} = ?$$

$$x^2 + y^2 = 25^2$$

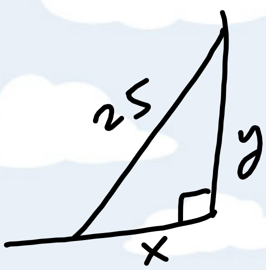
$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$$

$$(15)(3) + (20)\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-\cancelto{3}{15}\cdot 3}{\cancelto{4}{20}} = -\frac{9}{4}\ \frac{ft}{sec}$$

Ladder question continued:

At what rate is the area of the triangle formed by the wall, the ground and the ladder changing when the bottom of the ladder is 15 feet from the wall?



$\frac{dx}{dt} = 3$

$x = 15$

$y = 20$

$\frac{dy}{dt} = -\frac{9}{4}$

$\frac{dA}{dt} = ?$

$A = \frac{1}{2} x y$

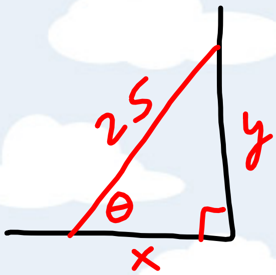
$$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt} \cdot y + \frac{1}{2} x \frac{dy}{dt}$$

$$= \frac{1}{2}(3)(20) + \frac{1}{2}(15)\left(-\frac{9}{4}\right)$$

$$= 30 + \frac{-135}{8} = \frac{105}{8}\ \frac{ft^2}{sec}$$

Ladder question continued:

At what rate is the angle formed by the ladder and the ground changing when the bottom of the ladder is 15 feet from the wall?



$\frac{dx}{dt} = 3$

$\frac{dy}{dt} = -\frac{9}{4}$

$x = 15$

$y = 20$

$\frac{d\theta}{dt} = ?$

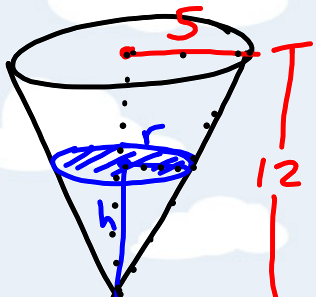
$$\sin\theta = \frac{y}{25} = \frac{1}{25}y$$

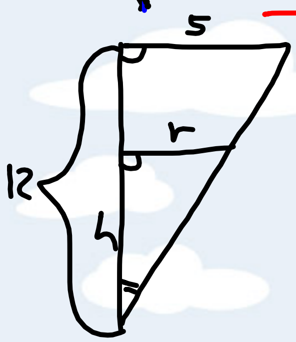
$$\cos\theta \frac{d\theta}{dt} = \frac{1}{25}\frac{dy}{dt}$$

$$\left(\frac{15}{25}\right)\frac{d\theta}{dt} = \frac{1}{25}\left(-\frac{9}{4}\right)$$

$$\frac{d\theta}{dt} = \frac{1}{\cancel{25}}\left(-\frac{\overset{3}{\cancel{9}}}{4}\right)\left(\frac{\cancel{25}}{\underset{5}{\cancel{15}}}\right) = \frac{-3}{20}\ \frac{\text{radians}}{\text{sec}}$$

ex) A conical tank (vertex down) is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of water when the water is 8 feet deep.





$\frac{dV}{dt} = 10$

$\frac{dh}{dt} = ?$

$h = 8$

$\frac{5}{r} = \frac{12}{h}$

$5h = 12r$

$r = \boxed{\frac{5}{12}h}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{5}{12}h\right)^2 h$

$V = \frac{1}{3}\pi \cdot \frac{25}{144} h^3$

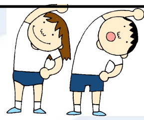
$\frac{dV}{dt} = \cancel{\frac{1}{3}}\pi \frac{25}{144} \cdot \cancel{3}h^2 \frac{dh}{dt}$

$10 = \pi \cdot \frac{25}{144}(8)^2 \frac{dh}{dt}$

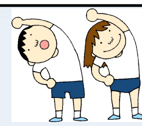
$\frac{dh}{dt} = \frac{10 \cdot 144}{25 \cdot 64 \cdot \pi}$ $\frac{ft}{min}$

ex) A spherical balloon is being inflated at a rate of 10 cubic centimeters per second. Find the rate of change of the surface area of the balloon at the moment when the surface area is 64π. (Note: $S = 4\pi r^2$ $V = (4/3)\pi r^3$)

(Hint: start by differentiating both equations separately and see if you can figure out a way to substitute in order to find dS/dt)



WARM UP!!



Given $3xy^2 = 12$

a) Find dy/dx

b) Find the equation of the line, in slope-intercept form, tangent to the curve at the point (1, 2)

c) Find the value of d^2y / dx^2 at the point (1, 2)

d) There is another point on the curve when x = 1. Find the slope of the tangent line at that point.

a) dy/dx = -y/2x

b) y = -x + 3

c) 3/2

d) point is (1, -2), slope is 1

What have we learned?

- Can I set up a related rates problem correctly and use appropriate steps to solve it?