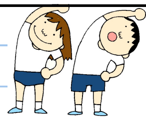
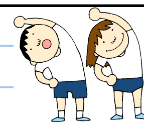


WARM UP/Review!!



Find dy/dx if:

a) $y = \cos(x^2 - 5)$

b) $y = (4x^2 - 3x)^6$

c) $y = \sin(\cos x)$

d) $y = (1 + (2x + 1)^4)^3$

e) $y = \cos^2(5x)$ $= [\cos(5x)]^2$

f) $y = \sin^5(\tan 3x)$ $= [\sin(\tan 3x)]^5$

$y' = 5[\sin(\tan 3x)]^4 \cdot \cos(\tan 3x) \cdot \sec^2(3x) \cdot 3$

$= 15\sin^4(\tan 3x)\cos(\tan 3x) \cdot \sec^2(3x)$

d) $y' = 3(1+(2x+1)^4)^2 \cdot 4(2x+1)^3(2)$

$= 24(1+(2x+1)^4)^2(2x+1)^3$

a) $y' = -2x\sin(x^2 - 5)$ b) $y' = 6(4x^2 - 3x)^5(8x - 3)$

c) $y' = -\sin x \cdot \cos(\cos x)$ d) $y' = 24(1 + (2x + 1)^4)^2(2x + 1)^3$

e) $y' = -10\cos(5x)\sin(5x)$ or $y' = -5\sin(10x)$

f) $y' = 15\sin^4(\tan 3x) \cdot \cos(\tan 3x) \cdot \sec^2(3x)$

f) $y = \sin^5(\tan 3x) = [\sin(\tan 3x)]^5$

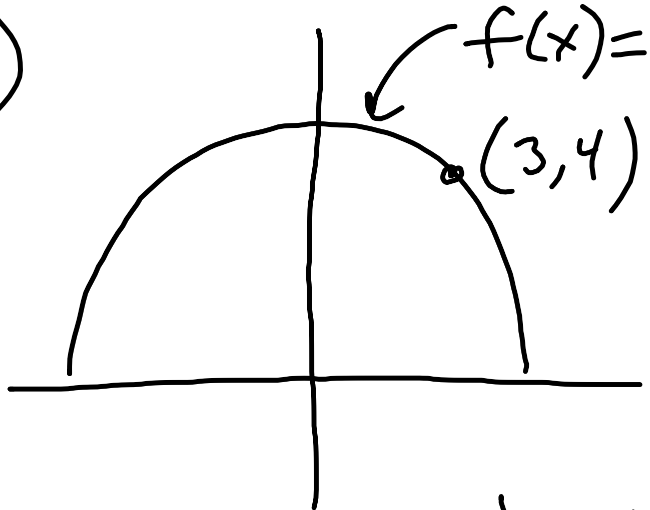
$$\frac{dy}{dx} = 5[\sin(\tan 3x)]^4 \cdot \cos(\tan 3x) \cdot \sec^2(3x) \cdot 3$$

d) $y = (1 + (2x+1)^4)^3$

$$\frac{dy}{dx} = 3[1 + (2x+1)^4]^2 \cdot 4(2x+1)^3 \cdot 2$$

HW 79

$f(x) = \sqrt{25 - x^2}$



$$f'(x) = \cancel{\tfrac{1}{2}}(25 - x^2)^{-\frac{1}{2}}(-\cancel{2}x)$$

$$= \frac{-x}{\sqrt{25 - x^2}} \qquad y - 4 = \frac{-3}{4}(x - 3)$$

$$f'(3) = \frac{-3}{\sqrt{16}} = \frac{-3}{4}$$

73 $f(x) = \tan^2 x$ $\quad \left(\frac{\pi}{4}, 1\right)$

$f'(x) = 2\tan x \cdot \sec^2 x$

$f'\left(\frac{\pi}{4}\right) = 2(1)(2) = 4$

$y - 1 = 4\left(x - \frac{\pi}{4}\right)$

$y = \sin ax$

$y' = a\cos ax$

39 a) $y = \sin x$

$y' = \cos x$

$y'(0) = \cos(0) = 1$

b) $y = \sin 2x$

$y' = 2\cos 2x$

$y'(0) = 2(1) = 2$

2.5 Implicit Differentiation

ESSENTIAL LEARNING TARGETS

At the end of this lesson, you will be able to:

- Differentiate implicitly defined expressions and understand how the chain rule relates to implicit differentiation

Explicit (solved for y) **Implicit** (not solved for y)

$y = -3x + 5$ $3x + y = 5$

$\frac{dy}{dx} = -3$ $3 + \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -3$

How to think about it

Think: $\frac{d}{dx}y = \frac{dy}{dx}$

In other words, the derivative of y with respect to x is dy/dx

Not so simple explanation: whenever we differentiate a variable that is not the indicated independent variable (the variable with which we are differentiating with respect to), we must multiply by the differential (the derivative of the new variable).

Think of chain rule: Suppose f(x) = y, find f '(x).

Think of this as f(x) = $(y)^1$.

$$f'(x) = 1(y)^0 \frac{dy}{dx} = \frac{dy}{dx}$$

$$f(x) = (y)^2$$

$$f'(x) = 2(y)^1 \frac{dy}{dx} = 2y\frac{dy}{dx}$$

$$f(x) = \sin(y)$$

$$f'(x) = \cos y \frac{dy}{dx}$$

$$f(x) = \sin(x)$$

$$f'(x) = \cos(x)\frac{dx}{dx}$$

$$= \cos(x)$$

Simple (or not so simple) examples.

Differentiate each of the following with respect to x and solve for dy/dx.

Then differentiate again with respect to t (don't solve for anything).

$\frac{d}{dt}$

a) $3y^2 = 4x$

$6y \cdot \frac{dy}{dx} = 4$

$\frac{dy}{dx} = \frac{4}{6y} = \frac{2}{3y}$

$\frac{d}{dt}$

$6y \frac{dy}{dt} = 4 \frac{dx}{dt}$

b) $8x^5 - 7(2y - 4)^3 = 8$

$40x^4 - 21(2y-4)^2\left(2\frac{dy}{dx}\right) = 0$

$-21(2y-4)^2(2)\frac{dy}{dx} = -40x^4$

$\frac{dy}{dx} = \frac{-40x^4}{-42(2y-4)^2} = \frac{20x^4}{21(2y-4)^2}$

$40x^4 \frac{dx}{dt} - 21(2y-4)^2\left(2\frac{dy}{dt}\right) = 0$

You try!

ex) Find dy/dx if $x^2 - 2y^3 + 4y = 2$

$$2x - 6y^2\frac{dy}{dx} + 4\frac{dy}{dx} = 0$$

$$-6y^2\frac{dy}{dx} + 4\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx}(-6y^2+4) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{4-6y^2} = \frac{2x}{6y^2-4}$$

ex) Differentiate $(3x)(y^3) - 2y = 7$ with respect to x, then solve for dy/dx

$$3y^3 + 3x \cdot 3y^2\frac{dy}{dx} - 2\frac{dy}{dx} = 0$$

$$9xy^2\frac{dy}{dx} - 2\frac{dy}{dx} = -3y^3$$

$$\frac{dy}{dx}(9xy^2-2) = -3y^3$$

$$\frac{dy}{dx} = \frac{-3y^3}{9xy^2-2} = \frac{3y^3}{2-9xy^2}$$

You try again!

Differentiate $y^2 = 5x$ with respect to t

$$2y\frac{dy}{dt} = 5\frac{dx}{dt}$$

Let's do a bunch!! (Just differentiate, don't solve for anything.)

1. $\frac{d}{dx}(3x)$
2. $\frac{d}{dx}(3y)$
3. $\frac{d}{dy}(3x)$
4. $\frac{d}{d\theta}(3x)$
5. $\frac{d}{d\theta}(3\theta)$
6. $\frac{d}{dx}(x^2)$
7. $\frac{d}{d\theta}(x^2)$
8. $\frac{d}{dx}(x^2 + y^2)$
9. $\frac{d}{d\theta}(x^2 + y^2)$
10. $\frac{d}{dy}(x^2 + y^2)$
11. $\frac{d}{dx}(xy^2)$
12. $\frac{d}{d\theta}(xy^2)$
13. $\frac{d}{dy}(xy^2)$
14. $\frac{d}{dx}(x + x^2 + y^3 + yx^{10})$

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1. $\frac{d}{dx}(3x) = 3$

2. $\frac{d}{dx}(3y) = 3\frac{dy}{dx}$

3. $\frac{d}{dy}(3x) = 3\frac{dx}{dy}$

4. $\frac{d}{d\theta}(3x) = 3\frac{dx}{d\theta}$

5. $\frac{d}{d\theta}(3\theta) = 3$

6. $\frac{d}{dx}(x^2) = 2x$

7. $\frac{d}{d\theta}(x^2) = 2x\frac{dx}{d\theta}$

8. $\frac{d}{dx}(x^2 + y^2) = 2x + 2y\frac{dy}{dx}$

9. $\frac{d}{d\theta}(x^2 + y^2) = 2x\frac{dx}{d\theta} + 2y\frac{dy}{d\theta}$

10. $\frac{d}{dy}(x^2 + y^2) = 2x\frac{dx}{dy} + 2y$

11. $\frac{d}{dx}(xy^2) = 2xy\frac{dy}{dx} + y^2$

12. $\frac{d}{d\theta}(xy^2) = 2xy\frac{dy}{d\theta} + y^2\frac{dx}{d\theta}$

13. $\frac{d}{dy}(xy^2) = 2xy + y^2\frac{dx}{dy}$

14. $\frac{d}{dx}(x + x^2 + y^3 + yx^{10}) = 1 + 2x + 3y^2\frac{dy}{dx} + 10x^9y + x^{10}\frac{dy}{dx}$

Great AP Practice problem: 2015 #6

What have we learned?

- Do I know the difference between implicitly and explicitly defined functions?

- Can I differentiate implicitly with respect to a variety of variables?

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