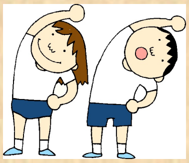
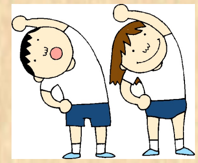


WARM UP!!



A theatre group is sending out letters to patrons asking for financial support. They have formulated that if x letters are sent, the amount of money donated (in thousands of dollars) will be $f(x) = 1/2x^2 + 14x + 90$. (These are very wealthy patrons.)

a) Find and interpret f (10) = 280

If 10 letters are sent, then \$280,000 will be donated

b) Express the rate of change in money donated with respect to the number of letters sent

$f'(x) = x + 14$

c) Find and interpret f '(10) = 24

If 10 letters are sent, then the amount of money donated will be changing at a rate of $\frac{\$24000}{\text{letter}}$

2.4a Chain Rule

ESSENTIAL LEARNING TARGETS

At the end of this lesson, you will be able to:

- apply the chain rule to differentiate composite functions
- know how to compute an average rate of change over an interval
- know that the derivative is the instantaneous rate of change of a function
- interpret the meaning of a derivative within the context of a problem using correct units
- express information about rates of change in applied contexts

Chain Rule: If y = f(g(x)), then y' = f '(g(x))· g'(x)

(Think, outside in.)

ex) $y = (3x + 2)^5$, find y'

$f(g(x)) = (3x+2)^5$

$f(x) = x^5$

$g(x) = 3x+2$

$y' = 5(3x+2)^4(3)$

ex) $f(x) = \dfrac{8x-3}{(5x+4)^7}$, find f '(x)

$$f'(x) = \frac{8(5x+4)^7 - (8x-3)(7)(5x+4)^6(5)}{(5x+4)^{14}}$$

$$= \frac{8(5x+4)^7 - (280x-105)(5x+4)^6}{(5x+4)^{14}}$$

$$= \frac{8(5x+4) - 280x + 105}{(5x+4)^8} = \frac{-240x+137}{(5x+4)^8}$$

You Try!

a) Find all values of x on the graph for which f '(x) = 0 and for which f '(x) is undefined.

$$f(x) = \sqrt[3]{(x^2-1)^2}$$

$$f(x) = (x^2-1)^{\frac{2}{3}}$$

$$f'(x) = \frac{2}{3}(x^2-1)^{-\frac{1}{3}}(2x)$$

$$= \frac{4x}{3\sqrt[3]{x^2-1}}$$

$$\boxed{\begin{array}{c} f'=0 \\ 4x=0 \\ x=0 \end{array}}$$

$$\boxed{\begin{array}{l} f' \text{ is undefined} \\ 3\sqrt[3]{x^2-1}=0 \\ x^2-1=0 \quad x=\pm 1 \end{array}}$$

b) Find the equation of the line tangent to y at t = -3

$$y = \frac{4}{(t+2)^2} = 4(t+2)^{-2}$$

point $(-3, 4)$

a) -5 b) infinity c) -5 d) DNE e) 1 f) 1 g) 4

h) 1 i) -2 j) -2 k) und l) -2

$$y' = \frac{-8}{(t+2)^3}$$

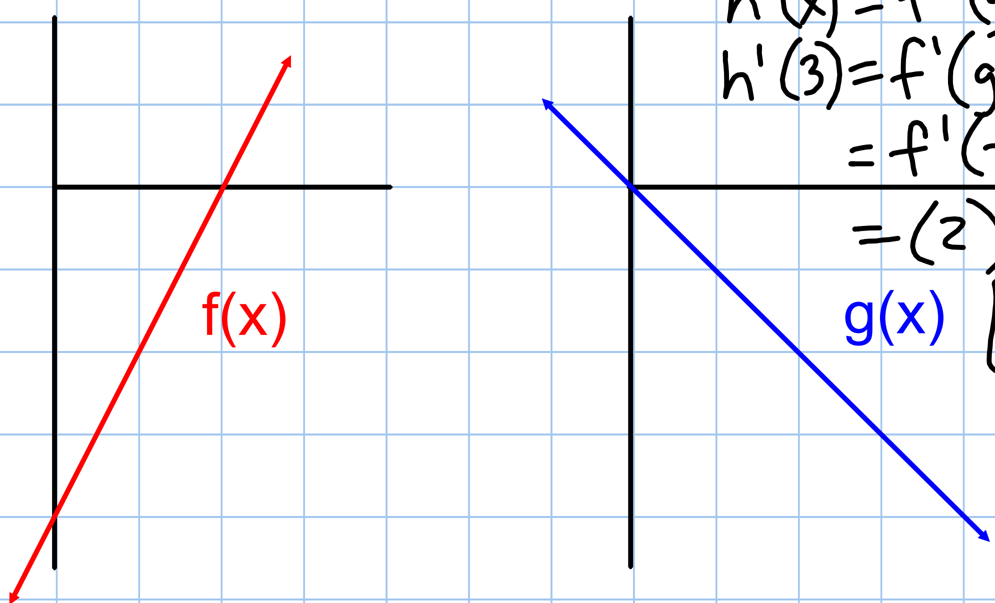
slope $= y'(-3) = 8$

$$\boxed{y-4=8(x+3)}$$

How well do you know the chain rule?

ex) Given the graphs below, find h'(3) if h(x) = f (g(x))

$$h'(x) = f'(g(x)) \cdot g'(x)$$
$$h'(3) = f'(g(3)) \cdot g'(3)$$
$$= f'(-3) \cdot g'(3)$$
$$= (2)(-1)$$
$$\boxed{= -2}$$



Now for something different

The function $y = ax^3 + bx^2 + cx + d$ is tangent to the line $y = 2x + 11$ at $(-2, 7)$ and is also tangent to the line $y = -4x + 3$ at $(0, 3)$. Find a, b, c, and d.

plug in $(-2,7)$: $7 = -8a + 4b - 2c + d$

plug in $(0,3)$: $3 = d$

$y' = 3ax^2 + 2bx + c$

plug $x = -2, y' = 2$: $2 = 12a - 4b + c$

when $x = 0, y' = -4$: $-4 = c$

$7 = -8a + 4b + 8 + 3$

so $-4 = -8a + 4b$

$6 = 12a - 4b$

$2 = 4a$

$a = \frac{1}{2}$ $b = 0$

$2 = 12a - 4b - 4$

$\boxed{y = \frac{1}{2}x^3 - 4x + 3}$

What have we learned?

- Can I state the chain rule in terms of f(x) and g(x)?
- Can I recognize when to apply the chain rule?
- Can I apply the chain rule in a variety of problems and situations?

September 22, 2016

September 22, 2016