

**WARMUP!!** Can you figure this out?

ex) Find all x-values at which  $f(x) = x^4 - x$  has a horizontal tangent line.

$$f'(x) = 4x^3 - 1 = 0$$

$$x^3 = \frac{1}{4}$$

$$x = \sqrt[3]{\frac{1}{4}}$$

ex) Find all x-values at which  $f(x) = x^3 - 6x + 5$  has a horizontal tangent line.

$$f'(x) = 3x^2 - 6 = 0$$

$$x^2 = 2$$

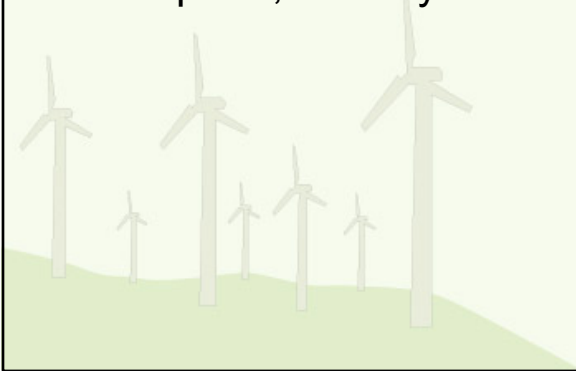
$$x = \pm\sqrt{2}$$

# Applications!!

## ESSENTIAL LEARNING TARGETS

At the end of this lesson, you will be able to:

- solve problems involving the slope of a tangent line
- solve rectilinear motion problems involving position, speed, velocity and acceleration



## Let's grow your brain!

ex) Suppose  $y = 8x + 1$  is tangent to the function  $f(x) = 8x^2 + 3$ .

Find the point of tangency.

$$f'(x) = 16x = 8$$

$$x = \frac{1}{2}$$

$$y\left(\frac{1}{2}\right) = 5$$

$$f\left(\frac{1}{2}\right) = 5$$

$$\boxed{\left(\frac{1}{2}, 5\right)}$$

ex) Suppose  $y = -4x + 7$  is tangent to the function  $f(x) = k - x^2$ .

Find k.

$$y' = -4$$

$$-2x = -4$$

$$x = 2$$

$$f'(x) = -2x$$

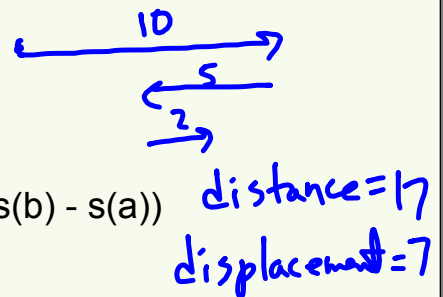
$$y(2) = -1$$

$$f(2) = k - 4 = -1$$

$$\boxed{k = 3}$$

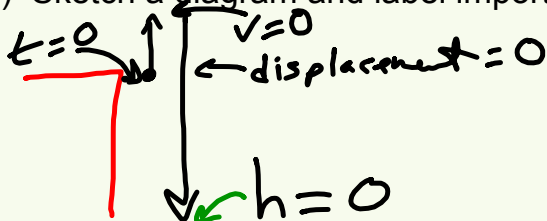
## Everything you ever wanted to know about PVA (position, velocity, acceleration) but were afraid to ask

1. Velocity is the derivative (rate of change) of position.
2. Acceleration is the derivative of velocity.
3. distance = total amount traveled
4. displacement = distance from starting position ( $s(b) - s(a)$ )
5. average velocity = displacement / time
6. a particle is at rest when the velocity = 0
7. motion is up/right when velocity  $> 0$ , down/left when velocity  $< 0$
8. speed = | velocity |
9. speed is increasing when velocity and acceleration have the same sign
10. speed is decreasing when velocity and acceleration have opposite signs



ex) A ball is thrown vertically upwards from the top of a tall building (right next to the building, so that it will eventually hit the ground). If the height of the ball at any given time,  $t \geq 0$ , is  $h(t) = -16t^2 + 64t + 80$ , answer the following:

- 1) Sketch a diagram and label important places <sup>h.a.</sup> 2) How tall is the building? <sup>S.</sup>



$$h(0) = 80 \text{ ft}$$

- 3) When does the ball reach maximum height? 4) What is the max height?

$$v(t) = h'(t) = -32t + 64 = 0$$

$$h(2) = 144 \text{ ft}$$

$$t = 2 \text{ sec}$$

- 5) How long does it take to hit the ground?

$$-16t^2 + 64t + 80 = 0 \quad t = 5, -1$$

$$t^2 - 4t - 5 = 0 \quad 5 \text{ sec}$$

$$(t - 5)(t + 1) = 0$$

- 7) What is the velocity at  $t = 1$ ? At  $t = 2$ ?

$$v(1) = 32 \text{ ft/sec}$$

$$v(2) = 0 \text{ ft/sec}$$

- 6) What was the initial velocity?

$$v(0) = 64 \frac{\text{ft}}{\text{sec}}$$

- 8) What is the height at  $t = 3$ ?

$$h(3) = 128 \text{ ft}$$

- 9) What is the speed when it hits the ground?

$$v(5) = -96$$

$$\text{speed} = 96 \text{ ft/sec}$$

- 10) What is the acceleration at

$t = 1$ ? At  $t = 2$ ?

$$a(t) = v'(t) = -32$$

$$a(1) = a(2) = -32 \frac{\text{ft}}{\text{sec}^2}$$

EX. A particle moves along the x-axis so that at any time,  $t$ , its position is given by  $x(t) = t^3 - 6t^2 + 9t + 11$  where  $t$  is measured in minutes and  $x(t)$  is measured in miles.

a) What is the velocity of the particle at  $t = 0$ ?

$$v(t) = x'(t) = 3t^2 - 12t + 9$$

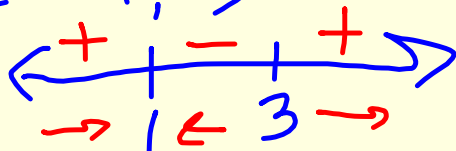
$$v(0) = 9 \frac{\text{miles}}{\text{min}}$$

b) During what time intervals is the particle moving to the left?

$$3t^2 - 12t + 9 = 0 \quad t = 1, 3$$

$$t^2 - 4t + 3 = 0$$

$$(t - 3)(t - 1) = 0$$



moving left on  $(1, 3)$  b/c  $v(t) < 0$

c) What is the acceleration of the particle at  $t = 2$ ?

$$a(t) = v'(t) = 6t - 12$$

$$a(2) = 0$$

AP Calculus AB 2007 Form B #3

parts a and b

(note: this is a calculator question)

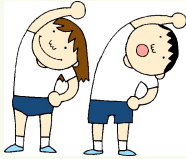


exam

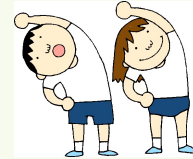


solutions





# Review!!



Given  $f(x) = 3x^2 - 2x + 4$ , find  $f'(x)$ . Then complete the table below:

point of tangency	slope at the point	tangent line equation
(0, 4)	-2	$y - 4 = -2x$ or $y = -2x + 4$
(-1, 9)	•	•
(3, 25)	•	•
(2, 12)	•	•
(a, f(a))	•	•
$x = 1$	•	•
$x = 1/2$	•	•



# What have we learned??

- What 2 things do we need to write the equation of a tangent line?
- How do we find the slope of a tangent line?
- What does PVA stand for?
- How does PVA relate to derivatives?

