

WARMUP!!

It's time to play, Name that Derivative!!!

For each function, let's see how many cells you need to see before you can figure out a general rule for the derivative

$$f(x) = x^2$$

x	f'(x)
1	2
2	4
3	6
4	8
5	10
x	2x

$$f(x) = x^3$$

x	f'(x)
1	3
2	12
3	27
4	48
5	75
x	$3x^2$

$$f(x) = x^4$$

x	f'(x)
1	4
2	32
3	108
4	256
5	500
x	$4x^3$

Based on these, can you find the pattern?

Power Rule: If $f(x) = x^n$, then $f'(x) = n x^{n-1}$

2.2a Derivative Shortcuts!!

ESSENTIAL LEARNING TARGETS

At the end of this lesson, you will be able to:

- calculate derivatives of selected functions including polynomial, power, rational and trigonometric
- apply appropriate derivative rules including the sum and difference rule



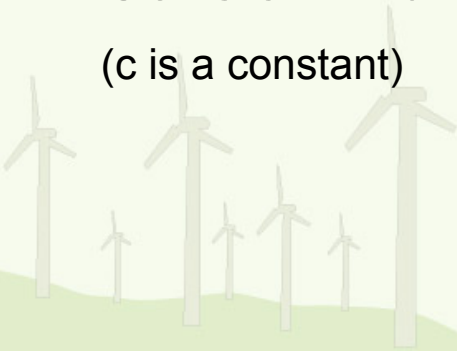
Constant Rule!

Suppose we graph a constant function, what does it look like? What would the slope be?



Constant Rule: If $y = c$, then $dy/dx = \bigcirc$

(c is a constant)



2 more rules:

Constant Multiple Rule: If $y = cf(x)$, then $y' = cf'(x)$

Sum/Difference Rule¹: If $y = f(x) \pm g(x)$,

$$\text{then } y' = f'(x) \pm g'(x)$$

Putting it together, ^{DNE} find y' for each below:

ex) $y = x^7$

$$y' = 7x^6$$

ex) $y = 5x^7$

$$y' = 35x^6$$

ex) $y = 5x^7 - 8x^3$

$$y' = 35x^6 - 24x^2$$

ex) $y = 5x^7 - 8x^3 + 12$

$$y' = 35x^6 - 24x^2$$

$$1) \underline{y} = x^{10} \quad y' = 10x^9$$

$$2) \underline{y} = 4x^3 \quad \frac{dy}{dx} = 12x^2$$

$$3) \underline{y} = \sqrt[3]{x} = x^{\frac{1}{3}} \quad y' = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$$

$$4) \underline{y} = \frac{4}{x^5} = 4x^{-5} \quad \frac{dy}{dx} = -20x^{-6} = \frac{-20}{x^6}$$

$$5) \underline{y} = 5x^8 - 3x^4 + 8x^{\frac{2}{3}} + 6x - \overset{x^{\frac{1}{5}}}{\sqrt[5]{x}} + \overset{2x^{\frac{1}{2}}}{\frac{2}{\sqrt{x}}} + 0.875$$

$$\begin{aligned} y' &= 40x^7 - 12x^3 + \frac{16}{3}x^{-\frac{1}{3}} + 6 - \frac{1}{5}x^{-\frac{4}{5}} - x^{-\frac{3}{2}} \\ &= 40x^7 - 12x^3 + \frac{16}{3\sqrt[3]{x}} + 6 - \frac{1}{5\sqrt[5]{x^4}} - \frac{1}{\sqrt{x^3}} \end{aligned}$$

That's all great for finding general derivatives, but what about derivatives at specific points?

ex) If $f(x) = x^{-2}$, find $f'(2)$

$$f'(x) = -2x^{-3} = -\frac{2}{x^3}$$

$$f'(2) = -\frac{2}{2^3} = -\frac{1}{4}$$

ex) Suppose $f(x) = \sqrt[3]{x^2}$, write the equation of the line tangent to $f(x)$ at $x = 8$

$$\rightarrow f(x) = x^{2/3}$$

$$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$$

$$f'(8) = \frac{2}{3\sqrt[3]{8}} = \frac{1}{3} \leftarrow \text{slope}$$

$$f(8) = \sqrt[3]{8^2} = 4 \quad (8, 4) \leftarrow \text{point}$$

\uparrow do the root first

$$\underline{y - 4 = \frac{1}{3}(x - 8)}$$

Let's talk Trig!!

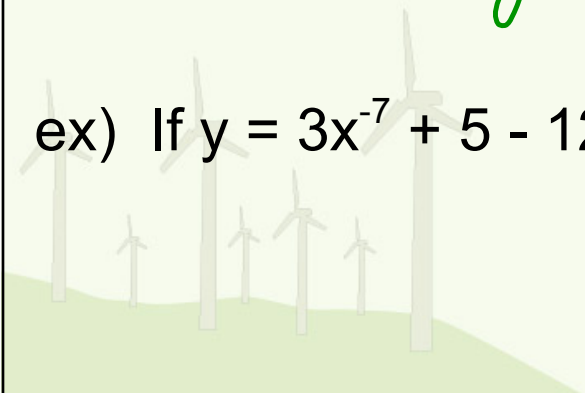
If $f(x) = \sin x$, then $\rightarrow f'(x) = \cos x$

If $f(x) = \cos x$, then $\rightarrow f'(x) = -\sin x$

ex) Find the derivative of $y = 8\sin x - 3\cos x$

$$y' = 8\cos x + 3\sin x$$

ex) If $y = 3x^{-7} + 5 - 12\sin x$, find dy/dx



Approximating the value of a derivative based on information presented in a table

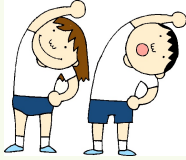
x	1	3	5	7	9
f(x)	4	8	10	3	-1

- 1) What is the average rate of change of $f(x)$ from $x = 1$ to $x = 9$?

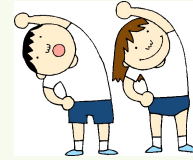
$$\text{aroc} = \frac{-1 - 4}{9 - 1} = \frac{-5}{8}$$

- 2) Estimate the instantaneous rate of change of $f(x)$ at $x = 5$.

$$\text{iroc} = \frac{3 - 8}{7 - 3} = \frac{-5}{4}$$



REVIEW!!



1. Without using the word 'derivative' or 'differentiation', write in words what is meant by $f'(2) = 4$
2. If $f(-6) = 10$ and $f'(-6) = 1/3$, write the equation of the line tangent to $f(x)$ at $x = -6$



What have we learned??

- What are the constant, power, constant multiple, and sum/difference rules for derivatives?
- What are the derivatives of $y = \sin x$ and $y = \cos x$?

