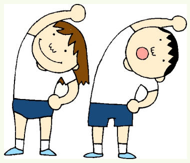
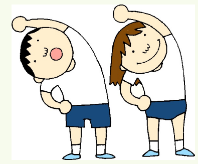
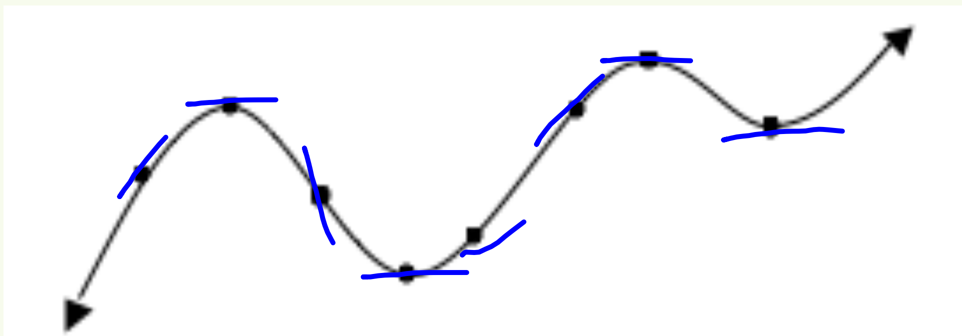


WARM UP!!



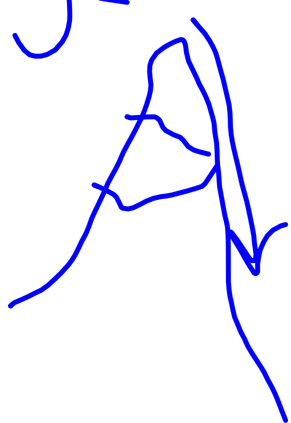


Sketch a small line tangent to the curve above at each of the indicated points.

Slope

$\frac{\Delta y}{\Delta x}$ rate of change $\frac{y_2 - y_1}{x_2 - x_1}$

$y = mx + b$



ski down a slope

2.1 Definition of the Derivative!!

ESSENTIAL LEARNING TARGETS

At the end of this lesson, you will be able to:

- know and apply the definition of a general derivative
- know and apply the definition of a derivative at a specific value of x
- express derivatives symbolically using the various methods of correct notation
- know the 3 conditions for differentiability and recognizes the connection between differentiability and continuity

Let's get Visual!

⭐ Let's see if we can generate (derive) a formula that will give us the slope of a tangent line.

NOTATION!!

Remember, the general derivative of f(x) is a: function whose output is the slope of the line tangent to f(x) at any given point

This can be written as:

derivative

verb

$f(x)$: $f'(x)$ or $\frac{d}{dx}f(x)$

y : y' or $\frac{dy}{dx}$ or $\frac{d}{dx}y$

noun

Quick Review: What 2 things do we need in order to write the equation of a tangent line?

[redacted] and [redacted]

3 FORMS of LINEAR EQUATIONS:

1) General: $ax + by = c$

2) Slope-Intercept: $y = mx + b$

3) Point-Slope: $y - y_1 = m(x - x_1)$

WHICH IS THE BEST FORM TO USE

IN THIS COURSE??? **<u>POINT-SLOPE FORM!!</u>**

Formulas!!!

General definition of the derivative
(gives a slope-finding function):

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

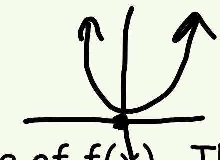
Specific (alternate) definitions of the derivative
(give the numerical slope at a specific value of x):

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

OR

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Let's try one together!



ex) If $f(x) = x^2$, find the general derivative of $f(x)$. Then find the specific derivative at $x = -5$ and use this to write the equation of the line tangent to $f(x)$ at $x = -5$.

$$f(x) = x^2$$

DNE

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$\boxed{f(x+h) = (x+h)^2}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= 2x$$

Slope at $x = -5$

$$f'(-5) = 2(-5) = -10$$

point

$$f(-5) = 25$$

$$\boxed{y - 25 = -10(x+5)}$$

You try!

ex) If $f(x) = \sqrt{x}$, find f'(x) and use this to find f'(9). Then write the equation of the line tangent to f(x) at x = 9.

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + h - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \boxed{\frac{1}{2\sqrt{x}}}$$

slope

$$f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$

point

$$f(9) = \sqrt{9} = 3$$

$$y - 3 = \frac{1}{6}(x - 9)$$

Aren't we going to practice the other definitions?? How about right now??

ex) Use an alternate definition of the derivative to find f'(4) if $f(x) = x^2 - 5x + 1$. Then write the equation of the line tangent to f(x) at x = 4.

$$f'(4) = \lim_{x \to 4} \frac{x^2 - 5x + 1 - (4^2 - 5(4) + 1)}{x - 4}$$

$$= \lim_{x \to 4} \frac{x^2 - 5x + 4}{x - 4}$$

$$= \lim_{x \to 4} \frac{\cancel{(x-4)}(x-1)}{\cancel{x-4}} = 3$$

slope at x = 4

point

$f(4) = -3$

$$\boxed{y + 3 = 3(x - 4)}$$

How about this one?

ex) Use an alternate definition of the derivative to find f'(-2) if f(x) = 2/(x-5). Then write the equation of the line tangent to f(x) at x = -2.

$$f'(-2) = \lim_{x \to -2} \frac{\frac{2}{x-5} - \frac{-2}{7}}{x+2} \cdot \frac{7(x-5)}{7(x-5)}$$

$$= \lim_{x \to -2} \frac{14 + 2x - 10}{(x+2)(7)(x-5)}$$

$$= \lim_{x \to -2} \frac{2x+4}{(x+2)(7)(x-5)}$$

$$= \lim_{x \to -2} \frac{2\cancel{(x+2)}}{\cancel{(x+2)}(7)(x-5)}$$

$$= \frac{-2}{49} \leftarrow \text{slope}$$

point

$$f(-2) = \frac{-2}{7}$$

$$\boxed{y + \frac{2}{7} = \frac{-2}{49}(x+2)}$$

⬆ Let's take it up a notch!

Can you work these problems backwards? For each expression below, rewrite the problem like this:

If f(x) = _______, find f'(________)

a) $\lim_{x\to 5} \frac{(8-x)^3 - 27}{(x-5)}$

$\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$f(x) = (8-x)^3$

$a = 5$

i.e. if $f(x) = (8-x)^3$

find $f'(5)$

b) $\lim_{\Delta x\to 0} \frac{\sqrt[3]{3+\Delta x} - \sqrt[3]{3}}{\Delta x}$

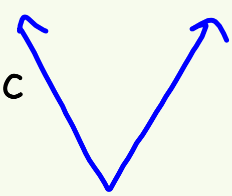
$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$f(x) = \sqrt[3]{x}$

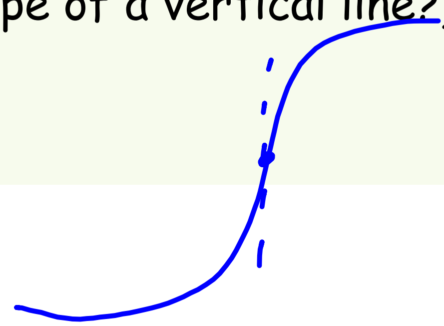
$a = 3$

3 Conditions for f(x) to be differentiable at x = c

1. f(x) must be continuous at x = c
2. f(x) cannot contain a sharp turn at x = c
   (i.e. $f'(x)^-$ must equal $f'(x)^+$)
   (Ponder, when could this happen?)



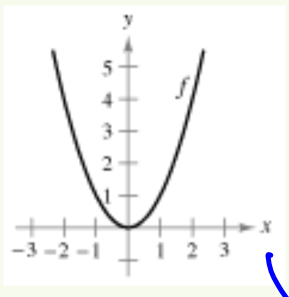
3. f(x) cannot have a vertical tangent at x = c
   (Ponder, can a function have a vertical tangent?)
   (Ponder, what is the slope of a vertical line?)



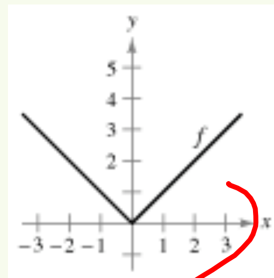
One more notch!!

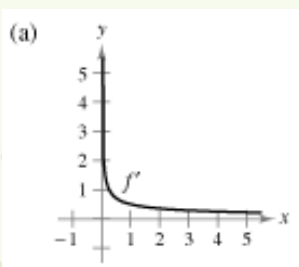
Match each graph below to the graph of its derivative

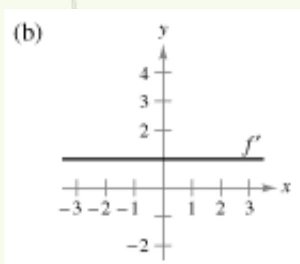
1

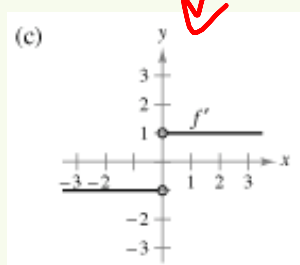


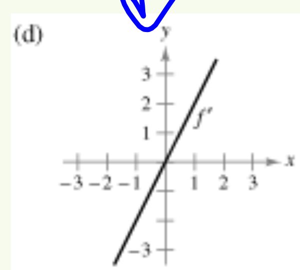
2











What have we learned??

- What is a derivative?
- What are the limit definitions use to determine general and specific derivative values?
- How do I use derivatives to write the equation of a line tangent to a curve?
- What are the 3 conditions that must be met for a function to be differentiable at a point?

September 16, 2016