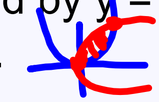
Warmup (review) (calculators permitted)

1) Given that $\rho = x^2 + y^2$ over the lamina bounded by $y = x^2$ and $x = y^2$, find the mass, first moments and centroid.



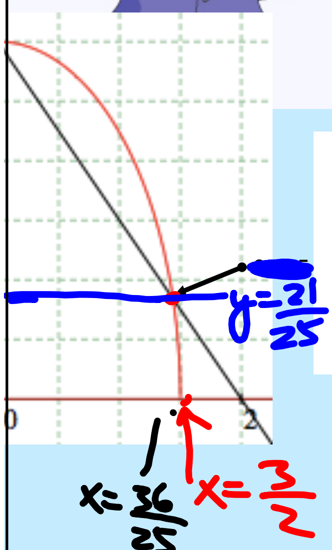
2) Find the volume in the first octant bounded by $z = 9 - 4x^2 - y^2$, $y = (-3/2)x + 3$ and the planes $x = 0$ and $y = 0$. (Be careful, the bounderies of the region in the xy-plane do not form a simple triangle. Remember, your region needs 4 boundaries.)

$$M_y = \int_0^1 \int_{x^2}^{\sqrt{x}} x(x^2 + y^2)dy\,dx \approx 0.109$$

$$M_x = \int_0^1 \int_{x^2}^{\sqrt{x}} y(x^2 + y^2)dy\,dx \approx 0.109$$

$$m = \int_0^1 \int_{x^2}^{\sqrt{x}} (x^2 + y^2)dy\,dx \approx 0.171$$

$$center\ of\ mass \approx (.637, .637)$$



To find the boundary, set $z = 9 - 4x^2 - y^2 = 0$ to get $y = \sqrt{9 - 4x^2}$

Then find the zero (x = 3/2) and point where this intersects with $y = -\frac{3}{2}x + 3$ $\left(x = \frac{36}{25}\right.$

$$V = \int_0^{\frac{36}{25}} \int_0^{-\frac{3}{2}x+3} (9 - 4x^2 - y^2)\,dy\,dx + \int_{\frac{36}{25}}^{\frac{3}{2}} \int_0^{\sqrt{9-4x^2}} (9 - 4x^2 - y^2)dy\,dx \approx 14.926$$

$$y = -\frac{3}{2}\left(\frac{36}{25}\right) + 3 = -\frac{54}{25} + \frac{75}{25} = \frac{21}{25}$$

$$\int_0^{21/25} \int_0^{\sqrt{\frac{9-y^2}{4}}} (9 - 4x^2 - y^2)\,dx\,dy + \int_{\frac{21}{25}}^{3} \int_0^{-\frac{2}{3}(y-3)} z\,dx\,dy$$

15.5 Surface Area!!

Learning Target

- I can find the area of the upper surface of a solid

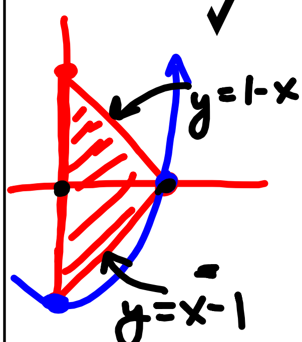
Remember arc length in 2 dimensions?

$$Arc\ length = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$$

Well, expanding this in 2 directions and taking it to 3 dimensions we get:

$$Surface\ area = \iint_R \sqrt{1 + [f_x(x,y)]^2 + \left[f_y(x,y)\right]^2}\,dA$$

ex) Write the integral that will give the area of the surface $f(x, y) = 1 - x^2 + y$ that lies above the triangular region with vertices (1, 0, 0), (0, -1, 0), and (0, 1, 0). Then use a calculator to evaluate the integral.



$$SA = \int_0^1 \int_{x-1}^{1-x} \sqrt{1 + (-2x)^2 + (1)^2}\, dy\, dx \approx 1.618$$

$$= \ln(\sqrt{3}+\sqrt{2}) + \frac{\sqrt{2}}{3}$$

In polar coordinates,

$$Surface\ area = \iint_R \sqrt{1 + [f_x(x,y)]^2 + \left[f_y(x,y)\right]^2}\ r\ dr\ d\theta\ (or\ r\ d\theta\ dr)$$

where $f_x(x,y)$ and $f_y(x,y)$ have been converted to r's and θ's

In polar coordinates,

$$Surface\ area = \iint_R \sqrt{1 + [f_x(x,y)]^2 + \left[f_y(x,y)\right]^2}\, r\, dr\, d\theta\ (or\ r\, d\theta\, dr)$$

where $f_x(x,y)$ and $f_y(x,y)$ have been converted to r's and θ's

ex) Find the surface area of the paraboloid $z = 1 + x^2 + y^2$ that lies above the unit circle.

$$f_x(x,y) = 2x\ \ and\ \ f_y(x,y) = 2y$$

$$\int_0^{2\pi}\int_0^{1} \sqrt{1 + 4x^2 + 4y^2}\, r\, dr\, d\theta = \int_0^{2\pi}\int_0^{1} \sqrt{1 + 4r^2}\, r\, dr\, d\theta$$

$$\int_0^{2\pi} \frac{1}{12}(1 + 4r^2)^{\frac{3}{2}}\Big|_0^1\, d\theta = \int_0^{2\pi} \frac{1}{12}(5\sqrt{5} - 1)d\theta$$

$$\frac{1}{12}(5\sqrt{5} - 1)\theta\Big|_0^{2\pi} = \frac{\pi}{6}(5\sqrt{5} - 1)$$

What have we learned?

- Can I find the area of the upper surface of a solid?