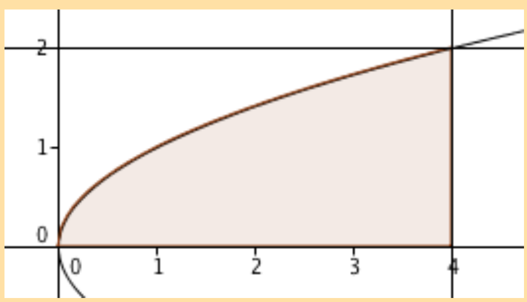
Warmup!

Sketch the region whose area is given by: $$\int_0^2 \int_{y^2}^4 dx\, dy$$

Then write a second iterated integral using the order dydx that would represent the same area.



$$\int_0^4 \int_0^{\sqrt{x}} dy\, dx$$

15.2 Order of Integration!!

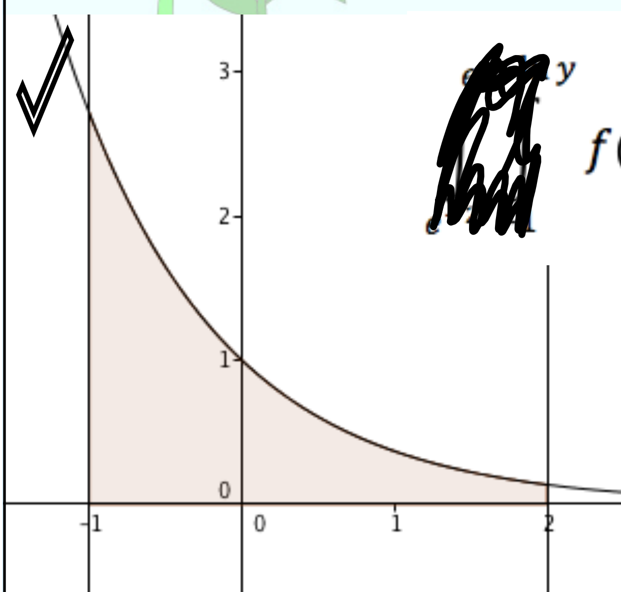
Learning Targets

- I can change the order of integration on an interated integral
- I can find the area of a plane region using multiple iterated integrals

There is no quick and easy method for reversing the order of integration, you just need to grunt it out. Just like chapter 7, often we might need to split the region into multiple integrals (like the one below). :)

ex) Sketch the region of integration for the iterated integral below and then switch the order of integration.

$$\int_{-1}^{2}\int_{0}^{e^{-x}} f(x,y)\,dy\,dx$$



[redacted] $f(x,y)\,dx\,dy + \int_{0}^{e^{-2}}\int_{-1}^{2} f(x,y)\,dx\,dy$

Lot of thrills with integration skills!

For each iterated integral below, find an equivalent integral by switching the order of integration, then evaluate the integrals using either form.

1) $$\int_0^3 \int_x^3 dy\, dx = \int_0^3 \int_0^y dx\, dy = \frac{9}{2}$$

2) $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\cos x} dy\, dx = 2\int_0^1 \int_0^{\arccos y} dx\, dy = 2$$

3) $$\int_0^4 \int_{\sqrt{y}}^2 x^2 y\, dx\, dy = \int_0^2 \int_0^{x^2} x^2 y\, dy\, dx = \frac{64}{7}$$

4) $$\int_0^2 \int_0^{e^x-1} 2(y-1) dy\, dx = \int_0^{e^2-1} \int_{\ln(y+1)}^2 2(y-1)\, dx\, dy = \frac{1}{2}e^4 - 4e^2 + \frac{19}{2}$$

5) $$\int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} r\, dr\, d\theta = \int_0^2 \int_0^{\arccos\left(\frac{r}{2}\right)} r\, d\theta\, dr = \frac{\pi}{2}$$

What have we learned?

- Can I sketch the region bounded by an interated integral?
- Can I switch the order of integration on an iterated integral?