Warmup!

1) Suppose $f_{xx}(a, b) = 9$, $f_{yy}(a, b) = 4$ and $f_{xy}(a, b) = 6$ and (a, b) is a critical point of $f(x, y)$. Does $f(x, y)$ have a relative max, min, or saddle point at (a, b), or is there insufficient info to determine this?

2) Ditto but $f_{xx}(a, b) = -9$, $f_{yy}(a, b) = 6$ and $f_{xy}(a, b) = 10$

✓

1) $d = 9(4) - 36 = 0$ so there is insufficient information

2) $d = -9(6) - 100 = -154$ so there is a saddle point at $(a, b, f(a, b))$

14.7b Absolute Extrema with Two Variables!

Learning Target:

- I can find absolute extrema for functions with two variables

To find absolute extrema of a surface bounded by a region, R on the xy-plane.

1) Find the critical points for the surface (where $f_x = 0$ and $f_y = 0$ and plug them into the surface equation to get the z-value(s).

2) Find the equation of intersection of each boundary with the surface equation.

3) Find the critical points for each intersection equation (using old fashioned derivatives) and test each of these by plugging them into the surface equation.

4) Test the 'corners' (intersection points for the boundaries) by plugging them into the surface equation.

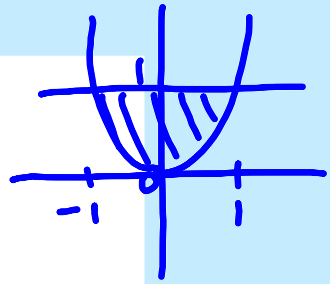
5) Largest z-value is the max, smallest z-value is the min

WARNING: Make sure the points you test (critical values for your surface and your boundaries) are within the bounded region!

ex) Find the absolute extrema of $f(x, y) = 2x - 2xy + y^2$ bounded by region R, which is the region in the xy-plane bounded by $\boxed{y = x^2}$ and $y = 1$.

$x^2 = 1$

$x = \pm 1$ $(-1, 1)$



1) Test the partials

$f_x(x, y) = 2 - 2y$ which = 0 at y = 1 $(1, 1)$

$f_y(x, y) = -2x + 2y$ which = 0 at y = x

So the critical point is (1, 1). $f(1,1) = 2 - 2 + 1 = \boxed{1}$

2) Test the boundaries

$$f(x, x^2) = 2x - 2x^3 + x^4$$

$$f'(x) = 2 - 6x^2 + 4x^3 = 0 \; so \; x = 1 \; or \; x = -\frac{1}{2}$$

$$f(1,1) = 1$$

$$f\left(-\frac{1}{2}, \frac{1}{4}\right) = -1 + \frac{1}{4} + \frac{1}{16} = \boxed{-\frac{11}{16}}$$

$$f(x, 1) = 2x - 2x + 1 = 1$$

3) Test the corners

$f(1,1) = \boxed{1}$, $f(-1, 1) = \boxed{1}$

So our max is 1 and our min is $-\frac{11}{16}$

If you graph the equation and the boundariers in grapher, change the appearance of the boundaries to be a solid color and zoom in so that it's easier to see.

ex) Find the absolute extrema for f(x,y) = sin(xy) on the closed region given by 0 ≤ x ≤ π and 0 ≤ y ≤ 1

$f_x = y\cos(xy)$

$f_y = x\cos(xy)$

$(0,0)$ $f(0,0) = \boxed{0}$

$\cos(xy) = 0$

$xy = \frac{\pi}{2}$

$f\left(xy = \frac{\pi}{2}\right) = \boxed{1}$

I know it's pretty obvious that the max is 1 and the min is 0, but let's walk through it anyway ☺

$f(x,y) = \sin(xy)$

1) Check out the critical points using the partials

$f_x(x,y) = y\cos(xy) = 0$ when $y = 0$ or $xy = \frac{\pi}{2}$

$f_y(x,y) = x\cos(xy) = 0$ when $x = 0$ or $xy = \frac{\pi}{2}$

Test $f(0,0) = 0, f\left(x, \frac{\pi}{2x}\right) = 1$

2) Find the equation of the intersection of each boundary with f(x, y) and find the relative extrema on each boundary

$f(0,y) = \sin 0 = 0$

$0 \le x \le \pi$

$f(x,0) = \sin 0 = 0$

$0 \le y \le 1$

$f(\pi,y) = \sin(\pi y)$, so $f' = \pi\cos(\pi y) = 0$ at $y = \frac{1}{2}$

$Test: f\left(\pi, \frac{1}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

$f(x,1) = \sin x$, so $f' = \cos x = 0$ at $x = \frac{\pi}{2}$

$Test: f\left(\frac{\pi}{2}, 1\right) = \sin\left(\frac{\pi}{2}\right) = 1$

3) Test the 'corners'

We've already tested $f(0,0), f(0,1), f(\pi,0)$

$f(\pi,1) = \sin\pi = 0$

So the absolute minimum of our function is $\boxed{0}$

and the absolute maximum of our function is $\boxed{1}$.

If you graph these in grapher, zoom way in to see it best. The x = π equation won't appear "attached", but it's still a pretty good image.

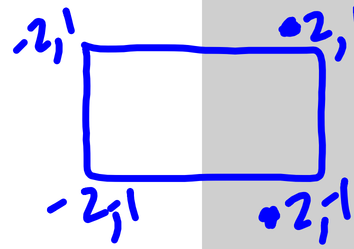
Find the absolute extrema for f(x, y) = $x^2 + 2xy + y^2$ over the region R = {(x,y): |x| ≤ 2, |y| ≤ 1} $-2 \le x \le 2$ $-1 \le y \le 1$

1) Test the partials

$f_x(x,y) = 2x + 2y \quad and \quad f_y(x,y) = 2x + 2y$

So both partial derivatives = 0 when $y = -x$

Test: $f(x,-x) = x^2 - 2x^2 + x^2 \boxed{= 0}$



2) Find the intersections of the boundaries with the surface and test each of them.

We can rewrite $|x| \le 2$ as $-2 \le x \le 2$ and $|y| \le 1$ as $-1 \le y \le 1$.
So the boundaries are $x = -2, x = 2, y = -1, y = 1$

$f(-2,y) = 4 - 4y + y^2$ so $f' = -4 + 2y = 0$ at $y = 2$
which is outside of our boundaries.

$f(2,y) = 4 + 4y + y^2$ so $f' = 4 + 2y = 0$ at $y = -2$
which is outside of our boundaries.

$f(x,-1) = x^2 - 2x + 1$, so $f' = 2x - 2 = 0$ at $x = 1$
$f(1,-1) = 0$

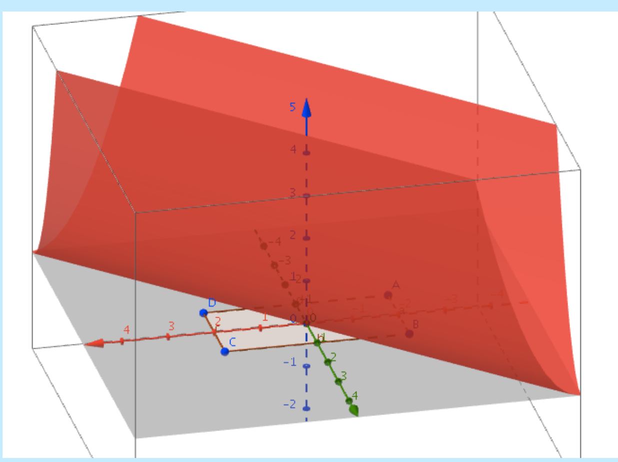
$f(x,1) = x^2 + 2x + 1$, so $f' = 2x + 2 = 0$ at $x = -1$
$f(-1,1) = 0$

3) Test the corners: (-2, -1), (2, -1), (-2, 1), (2, 1).

$f(-2,-1) \boxed{= 9,}$ $f(2,-1) \boxed{= 1,}$ $f(-2,1) \boxed{= 1,}$ $f(2,1) \boxed{= 9}$

So, our absolute maximum value is 9 and our absolute minimum value is 0.

Let's graph it and see what it looks like!



What have we learned?

- Can I find absolute extrema on a function of two variables?