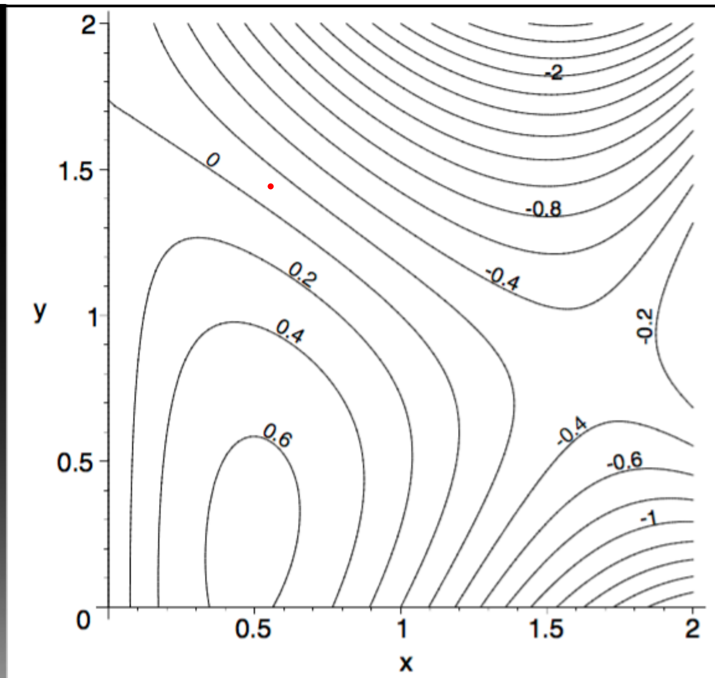


The graph to the left is a contour plot of a function f(x, y). The scale is 1 unit = 5 cm. The spacing between the contour levels is 0.2.

a) Use the contour plot to determine whether f_x and f_y are > 0, = 0, or < 0 at the point (1, 1.5).

If we hold y constant at 1.5, then move from left to right on either side of x = 1, f goes from -.4 to -.8, which would make $f_x < 0$.

✓

If we hold x constant at 1, then move from low to high on either side of y = 1.5, f goes again from -.4 to -.8, which would make $f_y < 0$ as well.

b) Use the contour plot to determine whether f_x and f_y are > 0, = 0, or < 0 at the point (1.2, 0.6).

If we hold y constant at 0.6, then move from left to right on either side of x = 1.2, f goes from 0 to -.2, which would make $f_x < 0$.

If we hold x constant at 1.2, then move from low to high on either side of y = 0.6, f goes again from -.2 to -.2, which would make $f_y = 0$.

✓

c) The function plotted on the figure is $f(x,y) = x^3 - xy^2 - 4x^2 + 3x + x^2y$

Calculate the actual values of the partial derivatives at (1, 1.5) and (1.2, 0.6).

✓

$$f_x = 3x^2 - y^2 - 8x + 3 + 2xy$$
$$f_y = -2yx + x^2$$

$$f_x(1, 1.5) = 3 - \frac{9}{4} - 8 + 3 + 3 = -\frac{5}{4}$$ ✓
$$f_y(1, 1.5) = -3 + 1 = -2$$ ✓

$$f_x(1.2, 0.6) = \frac{108}{25} - \frac{9}{25} - \frac{48}{5} + 3 + \frac{36}{25} = -\frac{6}{5}$$ ✓
$$f_y(1.2, 0.6) = 0$$ ☺ ✓

14.3b Partial Derivatives Continued

Learning Targets:

- I can find and apply partial derivatives for a function of multiple variables
- I can find second partial derivatives for a function of multiple variables

ex) Suppose $f(x, y) = 3xy^2 - 2y + 5x^2y^2$.

a) Find $f_x(x, y)$ and $f_y(x, y)$

$f_x(x, y) = 3y^2 + 10xy^2$

$f_y(x, y) = 6xy - 2 + 10x^2y$

b) Now find the second derivatives (see if you can figure out what the notation means)

$f_{xx}(x, y) = 10y^2$

$f_{xy}(x, y) = 6y + 20xy$

$f_{yx}(x, y) = 6y + 20xy$

$f_{yy}(x, y) = 6x + 10x^2$

What do you notice about f_{xy} and f_{yx}?

c) Now evaluate each second derivative at the point (-1, 2).

$f_{xx}(-1, 2) = 40$

$f_{xy}(-1, 2) = -28$

$f_{yx}(-1, 2) = -28$

$f_{yy}(-1, 2) = 4$

$f_{xy}(x, y)$ and $f_{yx}(x, y)$ are called "**mixed partial derivatives**", because they are partial derivatives with respect to mixed variables. As long as the derivatives are continuous, then the mixed partial derivatives with the same variables in any order will always be equal.

You try! Find the four second partial derivatives for $z = \sin(x - 2y)$

$z_x = \cos(x - 2y)$

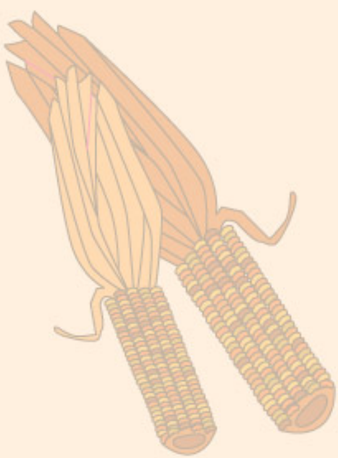
$z_y = -2\cos(x - 2y)$

$z_{xx} = -\sin(x - 2y)$

$z_{xy} = 2\sin(x - 2y)$

$z_{yx} = 2\sin(x - 2y)$

$z_{yy} = -4\sin(x - 2y)$



NOTATION

$$\frac{\partial}{\partial x} f(x,y) = f_x(x,y) = z_x = \frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial y} f(x,y) = f_y(x,y) = z_y = \frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = f_{xx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = f_{yy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} = f_{xy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = f_{yx}$$

Can you do these? This is a *whiteboard challenge*!

RULES:

1 problem per whiteboard.

Each problem may only be completed once.

Each group may only complete one problem (there are 4 total problems, so you may complete any of the 4).

A group may write a solution up on a whiteboard ONLY after it has been completely solved on paper. They may NOT solve as they write.

First group to get a problem completed on a whiteboard gets one point added to their next quiz grade. HOWEVER, if a group without a whiteboard finds a mistake, the original group forfeits the point and it goes to the group who corrects the mistake.

PROBLEMS:

Find all second partial derivatives for:

1) $f(x,y) = \ln\sqrt{xy} = \frac{1}{2}\ln xy = \frac{1}{2}(\ln x + \ln y)$

2) $z = \cos(x^2 + y^2)$

3) $f(x,y) = \sin(3x)\cdot\cos(3y)$

Find the first partial derivatives with respect to x, y and z for:

4) $g(x,y,z) = \dfrac{3xz}{x+y}$

1) $f_x(x,y) = \dfrac{y}{2xy} = \dfrac{1}{2x}$ $\quad f_y(x,y) = \dfrac{x}{2xy} = \dfrac{1}{2y}$

$f_{xx}(x,y) = -\dfrac{1}{2x^2}$ $\quad f_{yy}(x,y) = -\dfrac{1}{2y^2}$

$f_{xy}(x,y) = f_{yx}(x,y) = 0$

2) $\dfrac{\partial z}{\partial x} = -2x\sin(x^2+y^2)$ $\quad \dfrac{\partial z}{\partial y} = -2y\sin(x^2+y^2)$

$\dfrac{\partial^2 z}{\partial x^2} = -2\sin(x^2+y^2) - 4x^2\cos(x^2+y^2)$

$\dfrac{\partial^2 z}{\partial y^2} = -2\sin(x^2+y^2) - 4y^2\cos(x^2+y^2)$

$\dfrac{\partial^2 z}{\partial y\partial x} = \dfrac{\partial^2 z}{\partial x\partial y} = -4xy\cos(x^2+y^2)$

3) $f(x,y) = \sin(3x)\cdot\cos(3y)$

$f_x(x,y) = 3\cos(3x)\cdot\cos(3y)$

$f_y(x,y) = -3\sin(3x)\cdot\sin(3y)$

$f_{xx}(x,y) = -9\sin(3x)\cdot\cos(3y)$

$f_{yy}(x,y) = -9\sin(3x)\cdot\cos(3y)$

$f_{xy}(x,y) = f_{yx}(x,y) = -9\cos(3x)\cdot\sin(3y)$

4) $g(x,y,z) = \dfrac{3xz}{x+y}$

$g_x(x,y,z) = \dfrac{3z(x+y) - 3xz}{(x+y)^2} = \dfrac{3y}{(x^2+y^2)}$

$g_y(x,y,z) = \dfrac{-3xz}{(x+y)^2}$

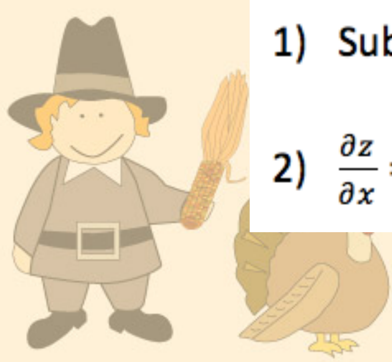
$g_z(x,y,z) = \dfrac{3x}{x+y}$

Do you remember?

Suppose the surface $f(x,y) = 9 - x^2 - y^2$ intersects the plane y = 2.

1) Find the equation of the curve of intersection.

2) Find the two slopes of the lines tangent to the curve of intersection at the points (2, 2, 1) and (-1, 2, 4) in the plane y = 2.



1) Substituting a 2 for y we get: $z = 5 - x^2$

2) $\frac{\partial z}{\partial x} = -2x$ so first slope = 2 and second slope = -4

What have we learned?

- Can I find and apply partial derivatives for a function of multiple variables?
- Can I find second partial derivatives for a function of multiple variables?