14.3a - Partial Derivatives!

Learning Targets:

At the end of this lesson, I can

- find and apply partial derivatives for a function of multiple variables

What is a partial derivative?

Suppose you are a chemist conducting an experiment, and you want to conduct it several times using varying amounts of the catalyst, while keeping other variables constant such as temperature and pressure.

This is similar to determining the rate of change of a function with respect to only one of its multiple independent variables.

This process is called partial differentiation and the result is the partial derivative of f with respect to the chosen independent variable.

You're going to love this.

Find the partial derivatives f_x and f_y for:

$$f(x,y) = 3x - x^2y^2 + 2x^3y$$

Try guessing these and see how you do.

$$f_x(x,y) = 3 - 2xy^2 + 6x^2y$$

$$f_y(x,y) = -2x^2y + 2x^3$$

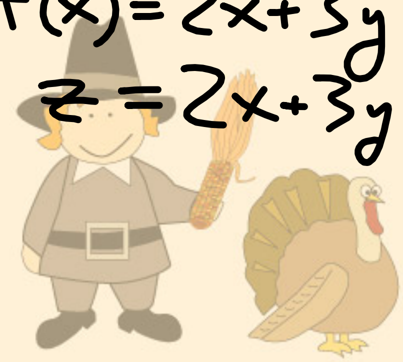
Notation!

$$\frac{\partial}{\partial x} f(x,y) = f_x(x,y) = z_x = \frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial y} f(x,y) = f_y(x,y) = z_y = \frac{\partial z}{\partial y}$$

$f(x) = 2x + 3y$

$z = 2x + 3y$



Note: ∂ is not the same as d.

∂ is a mathematical symbol that is often called 'del'.

Try these in your groups. Write the answers on your whiteboards.

1) Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x - y^2}{2x^2 + y}$

$$\lim_{(x,0)\to(0,0)} \frac{2x}{2x^2} = \lim_{(x,0)\to(0,0)} \frac{2}{4x} = DNE$$

$$\lim_{(0,y)\to(0,0)} \frac{-y^2}{y} = \lim_{(0,y)\to(0,0)} \frac{-2y}{1} = 0$$

because the limit does not approach the same value along different paths leading to (0, 0), the limit does not exist

2) Find the first partial derivatives of:

a) $z = \cos(x^2 + y^2)$

b) $f(x,y) = \dfrac{xy^2}{x^2 + y^2}$

a) $\dfrac{\partial z}{\partial x} = -2x\sin(x^2 + y^2)$ $\qquad \dfrac{\partial z}{\partial y} = -2y\sin(x^2 + y^2)$

b) $\dfrac{\partial}{\partial x} f(x,y) = \dfrac{y^2(x^2 + y^2) - xy^2(2x)}{(x^2 + y^2)^2} = \dfrac{y^4 - x^2y^2}{(x^2 + y^2)^2}$

$\dfrac{\partial}{\partial y} f(x,y) = \dfrac{2xy(x^2 + y^2) - xy^2(2y)}{(x^2 + y^2)^2} = \dfrac{2x^3y}{(x^2 + y^2)^2}$

3) Find all values of x and y such that:

$f_x(x,y) = 0 \quad and \quad f_y(x,y) = 0$

simultaneously if:

$f(x,y) = 3x^3 - 12xy + y^3$

$f_x(x,y) = 9x^2 - 12y = 0 \ \ so \ \ y = \frac{3}{4}x^2$

$f_y(x,y) = -12x + 3y^2 = 0$

$-12x + 3\left(\frac{3}{4}x^2\right)^2 = 0 \ \rightarrow \ -12x + \frac{27}{16}x^4 = 0$

$27x^4 - 192x = 0 \ \rightarrow \ 3x(9x^3 - 64) = 0$

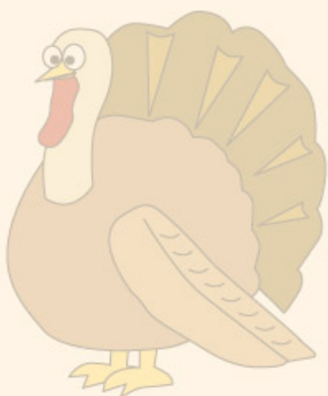
so $x = 0$ or $x = \dfrac{4}{\sqrt[3]{9}}$

and the points are: $(0,0)$ and $\left(\dfrac{4}{\sqrt[3]{9}}, \dfrac{3}{4}\left(\dfrac{4}{\sqrt[3]{9}}\right)^2\right) = \left(\dfrac{4}{\sqrt[3]{9}}, \dfrac{4}{\sqrt[3]{3}}\right)$

What does a partial derivative mean geometrically?

If you create a plane throuth the 'static' variable, then the value of the partial derivative evaluated at some specific point would be the slope of the line that lies on the plane, through the given point that is tangent to the curve.

The link demonstrates a line tangent to the curve that lies on the static plane of x = 1.
Note to me: rotate the view



Good video with graphics

Let's do some review!

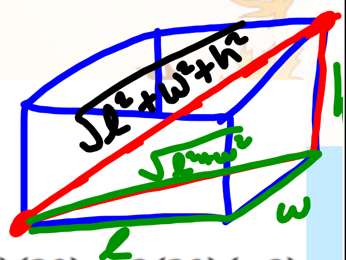
The length, l, width, w, and height, h, of a box all change with time. At a certain instant the dimensions are l = 10 ft, w = 20 ft and h = 20 ft. If, at this instant, l and w are increasing at a rate of 2 ft/sec while h is decreasing at a rate of 3 ft/sec, find the rates at which the following quantities are changing at this same instant:

a) volume

b) surface area

c) length of the diagonal

$$V = lwh \ \ so \ \ \frac{dV}{dt} = wh\frac{dl}{dt} + lh\frac{dw}{dt} + lw\frac{dh}{dt}$$
$$\frac{dV}{dt} = (20)(20)(2) + (10)(20)(2) + (10)(20)(-3)$$
$$= 600\frac{ft^3}{sec}$$



$$S = 2lw + 2lh + 2wh$$
$$\frac{dS}{dt} = 2\frac{dl}{dt}w + 2l\frac{dw}{dt} + 2\frac{dl}{dt}h + 2l\frac{dh}{dt} + 2\frac{dw}{dt}h + 2w\frac{dh}{dt}$$
$$\frac{dS}{dt} = 2(2)(20) + 2(10)(2) + 2(2)(20) + 2(10)(-3) + 2(2)(20) + 2(20)(-3)$$
$$= 80 + 40 + 80 - 60 + 80 - 120 = 100\frac{ft^2}{sec}$$

$$D = \sqrt{l^2 + w^2 + h^2} \ \ so \ \ D^2 = l^2 + w^2 + h^2$$
$$2D\frac{dD}{dt} = 2l\frac{dl}{dt} + 2w\frac{dw}{dt} + 2h\frac{dh}{dt}$$
$$2\sqrt{900}\frac{dD}{dt} = 2(10)(2) + 2(20)(2) + 2(20)(-3) = 0$$
$$\frac{dD}{dt} = 0\frac{ft}{sec}$$

What have we learned?

Can I:

- write a function of several variables using proper notation?
- find the domain and rage for a function of several variables?
- find the limit for a function of two variables?