

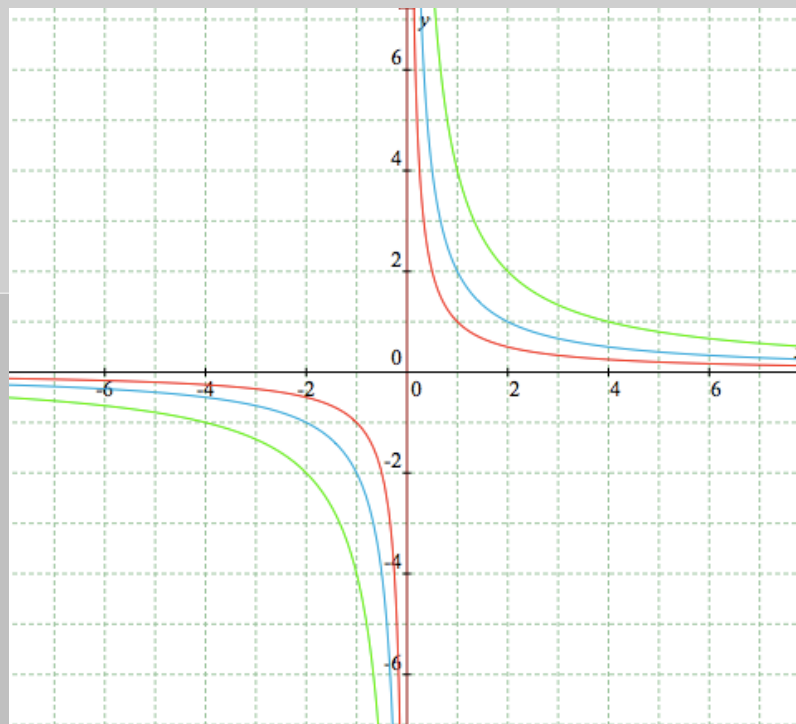


Warmup!

Use a 3d grapher to look at the graph of $f(x, y) = e^{\frac{1}{2}xy}$

Then sketch a level curve for the graph using $c = 1, e, e^2$, and $e^{1/2}$

Your general level curve equation would be $y = (2\ln c)/x$. Then just plug in each c and graph.



14.2 Limits of multiple variables

Learning Target:

- I can understand the concept of limits for a function of two variables





LIMITS!

When we did two-dimensional limits, the limit was the y -value that the function approached as x approached a specific value.

With three-dimensional limits, think of the limit as the z -value that a function approaches as a 'circle of space' closes in around a specific (x, y) ordered pair.

Note: if you need to show that a limit does not exist, you can show this by approaching the limit from 2 different 'paths' to show the limits are not the same:

ex) Show that the limit below does not exist.

$$\lim_{(x,y) \rightarrow (1,1)} \frac{x-1}{y-1}$$



$$\lim_{(x,x) \rightarrow (1,1)} \frac{x-1}{y-1} = \lim_{(x,x) \rightarrow (1,1)} \frac{x-1}{x-1} = 1$$

this would approach (1, 1) from the road of $y = x$

$$\lim_{(1,y) \rightarrow (1,1)} \frac{x-1}{y-1} = \lim_{(1,y) \rightarrow (1,1)} \frac{0}{y-1} = 0$$

this would approach (1, 1) from the road of $x = 1$

from what I have seen (and I'm new to this), these are easiest if you can find a value that causes the limit to = 1 and a different value that causes the limit to = 0

Can you figure these out?

Find the following:

$$1) \lim_{(x,y) \rightarrow (2,1)} (x + 3y^2) = 2 + 3 = 5$$

$$2) \lim_{(x,y) \rightarrow (1,1)} \left(\frac{x}{\sqrt{x+y}} \right) = \frac{1}{\sqrt{2}}$$

$$3) \lim_{(x,y) \rightarrow \left(\frac{\pi}{4}, 2\right)} y \cos(xy) = 2 \cos\left(\frac{\pi}{2}\right) = 0$$

$$4) \lim_{(x,y,z) \rightarrow (2,0,1)} x e^{yz} = 2e^0 = 2$$

$$5) \lim_{(x,y) \rightarrow (0,0)} \frac{1}{x^2 + y^2} = \infty \text{ (DNE)}$$

$$6) \lim_{(x,y) \rightarrow (0,0)} \frac{x}{x+y} = \text{DNE}$$

(because $\lim_{(x,y) \rightarrow (x,0)} \frac{x}{x+y} = 1$ but $\lim_{(x,y) \rightarrow (0,y)} \frac{x}{x+y} = 0$)

$$\lim_{(x,x) \rightarrow (0,0)} = \lim_{(x,x) \rightarrow (0,0)} \frac{x}{x+x} = \frac{1}{2}$$

$$\lim_{(0,y) \rightarrow (0,0)} = \lim_{(0,y) \rightarrow (0,0)} \frac{0}{y} = 0$$

What have we learned?

Can I find the limit for a function of two variables?

