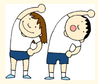
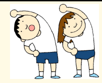


WARM UP!!



1) Prove that if an object is traveling at a constant speed, its velocity and acceleration vectors are orthogonal

2) Prove that an object moving in a straight line at constant speed has an acceleration of 0

3) The quarterback of a football team releases a pass at a height of 7 feet above the playing field, and the football is caught by a receiver 30 **yards** directly downfield at a height of 4 feet. The pass is released at an angle of 35˚ with the horizontal.

a) Find the speed of the football when it is released.

b) Find the maximum height of the ball

c) Find the time the ball is in the air

1) $r(t) = \langle x(t), y(t), z(t) \rangle$

$v(t) = \langle x'(t), y'(t), z'(t) \rangle$

$a(t) = \langle x''(t), y''(t), z''(t) \rangle$

speed = $\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}$ = constant

so d/dt (speed) = $2(x'(t))(x''(t) + 2(y'(t))(y''(t)) + 2(z'(t))(z''(t)) = 0$

$v(t) \cdot a(t) = (x'(t))(x''(t)) + (y'(t))(y''(t)) + (z'(t))(z''(t))$

but this equals (1/2) x d/dt (speed) = 0 so the dot product must equal 0 and the vectors are orthogonal

2) for the object to move in a straight line, its position components would all be linear

so $r(t) = \langle at + b, ct + d, et + f \rangle$

$v(t) = \langle a, c, e \rangle$

$a(t) = \langle 0, 0, 0 \rangle$

3) $h_0 = 7$, $\theta = 35˚$, passes through (90, 4) (30 yards is 90 ft)

a) $r(t) = \langle v_0(\cos 35˚)t, 7 + v_0(\sin 35˚)t - 16t^2 \rangle$

$v_0 \cos 35˚ t = 90$ so $t = 90 / (v_0 \cos 35˚)$

$7 + v_0(\sin 35˚)(90 / (v_0 \cos 35˚) - 16(90 / (v_0 \cos 35˚))^2 = 4$

so $v_0 \approx 54.09$ ft/sec

b) so $r(t) \approx \langle 44.307t, 7 + 31.024t - 16t^2 \rangle$

$v(t) = \langle 44.307, 31.024 - 32t \rangle$

$31.024 - 32t = 0$ at $t \approx 0.969496$ sec

max height $\approx y(0.969) \approx 22.04$ ft

c) $44.307t = 90$ so $t \approx 2.03$ sec

13.4b PVA again!

LEARNING TARGETS
At the end of this lesson, you will be able to:

- solve more pva-related problems
- get at least one quiz completed

1) A projectile is fired from ground level at an angle of 8° with the horizontal. Find the minimum velocity necessary if the projectile is to have a range of 50 meters.

2) Prove that if r(t) • r(t) equals a constant, then r(t) and r'(t) are orthogonal.

3) Given u = < -2, 7, -4 > and v = < -1, -2, 5 >, find $\text{proj}_u v$.

1) $r(t) = \langle (v_0\cos 8^\circ)t, (v_0\sin 8^\circ)t - 4.9t^2 \rangle$

$(v_0\cos 8^\circ)t = 50$ so $t = 50/(v_0\cos 8^\circ)$

$(v_0\sin 8^\circ)(50/v_0\cos 8^\circ) - 4.9(50/v_0\cos 8^\circ)^2 = 0$

$v_0 \approx 42.163$ m/sec

2) $r(t) = \langle x(t), y(t), z(t) \rangle$

$r(t) \bullet r(t) = (x(t))^2 + (y(t))^2 + (z(t))^2 = \text{constant}$

so $2(x(t))(x'(t)) + 2(y(t))(y'(t)) + 2(z(t))(z'(t)) = 0$

since $r(t) \bullet r'(t) = (x(t))(x'(t)) + (y(t))(y'(t)) + (z(t))(z'(t))$

this must equal 0

3) $\text{proj}_u v = [(2 - 14 - 20) / (\sqrt{69})^2] \langle -2, 7, -4 \rangle$

$= (32/69)\langle -2,7, 4 \rangle$

A bomber is flying horizontally at an altitude of 3200 feet with a velocity of 400 ft/sec when it releases a bomb. A projectile is launched 5 seconds later from a cannon at a site facing the bomber and 5000 feet from the point beneath the original position of the bomber. If the projectile is to intercept the bomb at an altitude of 1600 feet, determine the initial speed and angle of inclination of the projectile.

$r_b(t) = < 400\cos 0^\circ t,\ 3200 + 400\sin 0^\circ t - 16t^2 >$

$= < 400t,\ 3200 - 16t^2 >$

$3200 - 16t^2 = 1600$ at $t = 10$, so 10 seconds after the bomb is released, it will be at position $< 4000, 1600 >$

$r_p(t) = < v_0\cos\theta t,\ 0 + v_0\sin\theta t - 16t^2 >$

Since the projectile is launched 5 seconds later, our $t = 5$

we want $r_p(5) = < 1000, 1600 >$

so $5v_0\cos\theta = 1000$ and $v_0 = 200/\cos\theta$

$(200/\cos\theta)(\sin\theta)(5) - 80 = 1600$

$\theta \approx 63.43^\circ$

$v_0 = 200/\cos 63.43^\circ \approx 447.21$ ft/sec

What have we learned??

- Can I solve more pva-related problems?
- Can I get at least one quiz completed?

October 2, 2019