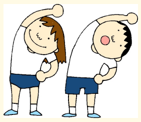
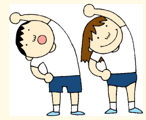


WARM UP!!



1) Given $r(t) = \langle 2\sin t, 2\cos t, 4\sin^2 t \rangle$

a) Find $T(t)$

b) Find a parametric set of equations of the line tangent to $r(t)$ at the point $P(1, \sqrt{3}, 1)$

2) Find $N(-\pi/4)$ for $r(t) = \langle \cos t, 2\sin t, 1 \rangle$

1) a) $r'(t) = \langle 2\cos t, -2\sin t, 8\sin t\cos t \rangle$

$$\|r'(t)\| = \sqrt{4\cos^2 t + 4\sin^2 t + 64\sin^2 t\cos^2 t}$$

$$= \sqrt{4 + 64\sin^2 t\cos^2 t}$$

$$T(t) = \frac{\langle 2\cos t, -2\sin t, 8\sin t\cos t \rangle}{\sqrt{4 + 64\sin^2 t\cos^2 t}} = \frac{\langle \cos t, -\sin t, 4\sin t\cos t \rangle}{\sqrt{1 + 16\sin^2 t\cos^2 t}}$$

b) The curve passes through the point at $t = \pi/6$, so the direction vector for the curve at $t = \pi/6$ would be $\langle \sqrt{3}, -1, 2\sqrt{3} \rangle$ and the equation would be: $x = 1 + \sqrt{3}t$, $y = \sqrt{3} - t$, $z = 1 + 2\sqrt{3}t$

2) $r'(t) = \langle -\sin t, 2\cos t, 0 \rangle$

$$\|r'(t)\| = \sqrt{(\sin^2 t + 4\cos^2 t)}$$

$$T(t) = \frac{\langle -\sin t, 2\cos t, 0 \rangle}{\sqrt{(\sin^2 t + 4\cos^2 t)}}$$

$$N(t) = \frac{\langle -2\cos t, -\sin t, 0 \rangle}{\sqrt{(\sin^2 t + 4\cos^2 t)}}$$

So $N(-\pi/4) = \dfrac{\langle -\sqrt{2}, \sqrt{2}/2, 0 \rangle}{\sqrt{(1/2 + 2)}} = \dfrac{\langle -2, 1, 0 \rangle}{\sqrt{5}}$

13.4a PVA again!

LEARNING TARGETS

At the end of this lesson, you will be able to:

- determine the velocity and acceleration associated with a vector-valued function
- use a vector-valued function to analyze projectile motion

If a position vector is written as r(t) = <x(t), y(t)>, then the velocity vector would be r'(t) = <x'(t), y'(t)> and the acceleration vector would be r''(t) = <x''(t), y''(t)>.

The **direction** of the velocity vector is actually the **direction** of motion of the object.

The **magnitude** of the velocity vector is actually the **speed** of the object.

Position: r(t) = <x(t), y(t)>

Velocity: v(t) = r'(t) = <x'(t), y'(t)>

Acceleration: a(t) = r''(t) = <x''(t), y''(t)>

Direction: arctan(y'(t)/x'(t))

Speed: $||v(t)|| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$

All of these can be expanded to 3 dimensions except for direction, which we'll talk about later.

ex) For $r(t) = <3\text{cos}t, 2\text{sin}t>$, find the velocity vector, acceleration vector and speed. Then sketch a graph of the path of the object as well as the velocity and acceleration vectors at the point (3, 0)

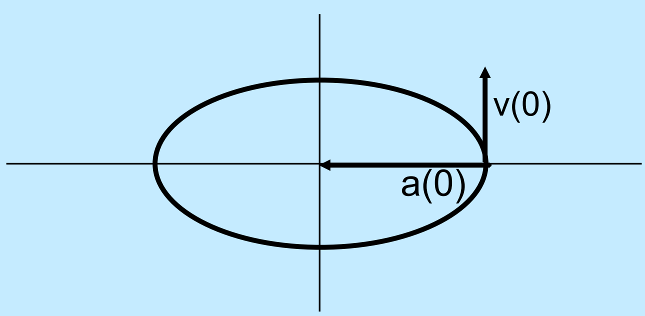
$v(t) = <-3\text{sin}t, 2\text{cos}t>$

$a(t) = <-3\text{cos}t, -2\text{sin}t>$

$\text{speed} = \sqrt{9\text{sin}^2t + 4\text{cos}^2t}$

We need to find the t-value that goes with the point (3, 0). So $3\text{cos}t = 3$ which means $t = 0$.

$v(0) = <0, 2>$ and $a(0) = <-3, 0>$



Suppose $a(t) = \langle 2, 0, 3 \rangle$.

Find $r(t)$ if $v(0) = \langle 0, 4, 0 \rangle$ and $r(0) = \langle 0, 0, 0 \rangle$.

$v(t) = \langle 2t + C_1, 0 + C_2, 3t + C_3 \rangle$

since $v(0) = \langle 0, 4, 0 \rangle$, $v(t) = \langle 2t, 4, 3t \rangle$

$r(t) = \langle t^2 + K_1, 4t + K_2, (3/2)t^2 + K_3 \rangle$

since $r(0) = \langle 0, 0, 0 \rangle$, $r(t) = \langle t^2, 4t, (3/2)t^2 \rangle$

Projectile Motion (without air resistance)

If h_0 = initial height, v_0 = initial velocity, θ = angle of elevation, and g = gravitational constant, the path of a projectile is:

$r(t) = \langle (v_0\cos\theta)t,\ h_0 + (v_0\sin\theta)t - (1/2)gt^2 \rangle$

A baseball is hit 3 feet above ground level at 100 ft/sec and at an angle of 45° with respect to the ground. Find the maximum height reached by the baseball. Will it clear a 10-foot-high fence located 300 feet from home plate?

$v_0 = 100,\ h_0 = 3$
$\theta = 45°$
$g = 32$

$r(t) = \langle 100\cos 45°t,\ 3 + (100\sin 45°)t - (1/2)(32)t^2 \rangle$

$= \langle 50\sqrt{2}\,t,\ 3 + 50\sqrt{2}\,t - 16t^2 \rangle$

For max height, we want the max of the y-component so set y' = 0 and solve for t to find the time when this occurs

$y'(t) = 50\sqrt{2} - 32t = 0$ at $t = 25\sqrt{2} / 16 \approx 2.2097$sec

$y(2.2097) = 3 + 50\sqrt{2}\,(2.2097) - 16(2.2097)^2 \approx 81.125$ ft

For the fence, first find the time: $50\sqrt{2}\,t = 300$, so $t \approx 4.2426$

$y(4.2426) \approx 3 + 50\sqrt{2}\,(4.2426) - 16(4.2426)^2 \approx 15$ ft

so the fence is safe :)

Let's dig a little deeper!

Given that the path of a projectile is:

$r(t) = \langle (v_0\cos\theta)t, h_0 + (v_0\sin\theta)t - (1/2)gt^2 \rangle$

Assuming h_0, v_0, θ and g are all constants, find the velocity and acceleration vectors based on r(t)

$v(t) = \langle v_0\cos\theta, v_0\sin\theta - gt \rangle$

$a(t) = \langle 0, -g \rangle$

So for any projectile motion situation, which way will the acceleration vector be pointing?

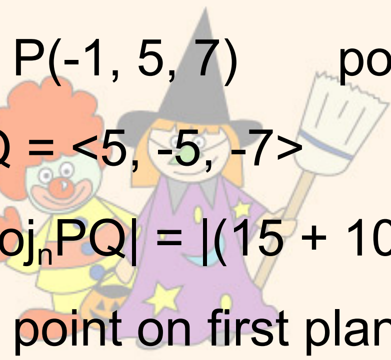
Always straight down!

Review!

1) Find the distance from point (-1, 5, 7) to plane $3x - 2y + 4z = 12$

2) Find the distance from plane $3x + y - z = 8$ to plane $3x + y - z = -3$

3) Find the distance from point (-2, 5, 5) to line: $x = 3 + 2t$, $y = -t$, $z = 1 - 4t$



1) P(-1, 5, 7)   point on plane: Q(4, 0, 0)

$PQ = \langle 5, -5, -7 \rangle$ $n = \langle 3, -2, 4 \rangle$

$|\text{proj}_n PQ| = |(15 + 10 - 28) / \sqrt{(29)}| \approx 0.557$

2) point on first plane: P(0, 8, 0)

point on second plane: Q(0, -3, 0)

$PQ = \langle 0, 11, 0 \rangle$ $n = \langle 3, 1, -1 \rangle$

$|\text{proj}_n PQ| = 11/\sqrt{11} \approx 3.316$

3) point P(-2, 5, 5)   point on line: Q(3, 0, 1)

$PQ = \langle 5, -5, -4 \rangle$ dir vector for line: $n = \langle 2, -1, -4 \rangle$

$\text{proj}_n PQ = (10 + 5 + 16) / \sqrt{(21)} \approx 6.765$

$||PQ|| = \sqrt{(66)} \approx 8.124$

$d^2 \approx 66 - (6.765)^2 \approx 20.235$ so distance ≈ 4.498

What have we learned??

- Can I describe the velocity and acceleration associated with a vector-valued function?
- Can I use a vector-valued function to analyze projectile motion?

October 1, 2019