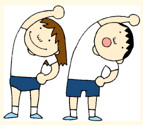
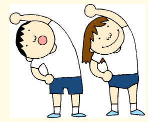


WARM UP!!



Find each of the following:

1) $\displaystyle\int \frac{3x}{x^2-3x-4}\,dx$

2) $\displaystyle\int 5x\cos x\,dx$

1) $\displaystyle\int \frac{3x}{x^2-3x-4}\,dx = \int\left(\frac{12}{5(x-4)}+\frac{3}{5(x+1)}\right)dx = \frac{12}{5}\ln|x-4| + \frac{3}{5}\ln|x+1| + C$

2) $\displaystyle\int 3\sqrt{4-x^2}\,dx = \int -12\sin^2\theta\,d\theta = \int -6(1-\cos 2\theta)\,d\theta$

$\displaystyle= -6\theta + 3\sin 2\theta + C = -6\arccos\frac{x}{2} + 6\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right) + C = -6\arccos\frac{x}{2} + \frac{3}{2}x\sqrt{4-x^2} + C$

(using a trig sub where $\sin\theta = \frac{\sqrt{4-x^2}}{2}$)

3) $\displaystyle\int 5x\cos x\,dx = 5x\sin x + 5\cos x + C$

12.2b Integrals of Vector-Valued Functions

LEARNING TARGETS

At the end of this lesson, you will be able to:

- integrate a vector-valued function

To antidifferentiate a vector-valued function, simply antidifferentiate each component separately. Each component will have its own unknown constant, but you can write the unknown constants vector as one vector identified as a bold capital **C**.

ex) $\int \langle t^3, \sin t, \frac{1}{t} \rangle \, dt = \langle \frac{1}{4}t^4, -\cos t, \ln|t| \rangle + \mathbf{C}$ ✓

$$= \langle \frac{1}{4}t^4, -\cos t, \ln|t| \rangle + \langle C_1, C_2, C_3 \rangle$$

$$= \langle \frac{1}{4}t^4 + C_1, -\cos t + C_2, \ln|t| + C_3 \rangle$$

Definite integrals of vector-valued functions work the same way as 'regular' functions. Just treat each component as a separate definite integral.

$$\int_0^1 \langle \sqrt[3]{t}, \frac{1}{t+1}, e^{-t} \rangle \, dt = \langle \frac{3}{4} t^{\frac{4}{3}}, \ln|t+1|, -e^{-t} \rangle \Big|_0^1 = \langle \frac{3}{4}, \ln 2, -\frac{1}{e} + 1 \rangle$$

ex) Find r(t), given that r ''(t) = <-4cost, 0, -3sint>, r '(0) = <0, 0, 3> and r(0) = <0, 4, 0>

r '(t) = <-4sint + C_1, 0 + C_2, 3cost + C_3>

since r '(0) = <0, 0, 3>,

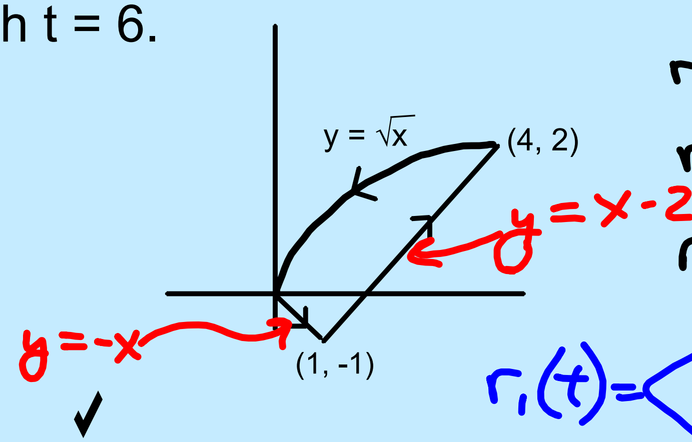
r '(t) = <-4sint, 0, 3cost>

r(t) = <4cost + K_1, 0 + K_2, 3sint + K_3>

since r(0) = <0, 4, 0>,

r(t) = <4cost - 4, 4, 3sint>

Review: Write a vector-valued function for the path of a particle shown below, starting at the origin with t = 0, so that each 'leg' of the journey takes 2 seconds, ending with t = 6.



$r_1(t) = \langle t, -t \rangle$, t in $[0, 1]$

$r_2(t) = \langle t, t-2 \rangle$, t in $[1, 4]$

$r_3(t) = \langle t, \sqrt{t} \rangle$, t in $[4, 0]$

$r_1(t) = \langle \frac{1}{2}t, -\frac{1}{2}t \rangle$, t in $[0, 2]$

$r_2(t) = \langle \frac{3}{2}t - 2, \frac{3}{2}t - 4 \rangle$, t in $[2, 4]$

$r_1(t) = < t/2, -t/2 >, 0 \le t \le 2$

$r_2(t) = <(3/2)t - 2, (3/2)t - 4>, 2 \le t \le 4$

$r_3(t) = <-2t + 12, \sqrt{-2t + 12} >, 4 \le t \le 6$

What have we learned??

- Can I find the integral of a vector-valued function?