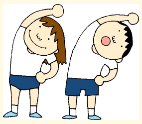
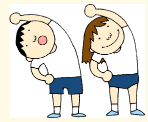
September 19, 2019



WARM UP!!



Find each of the following:

1) $\frac{d}{dx}(3x^2 - 2x + 6)$

2) $\frac{d}{dx}(5x\sqrt{x-6})$

3) $\frac{d}{dx}((5x^3 - 4x)^7)$

4) $\frac{d}{dx}\left(\frac{4x}{2x^2+3}\right)$

5) $\frac{d}{dx}(\sin 8x)$

6) $\frac{d}{dx}(3e^{4x})$

7) $\frac{d}{dx}(5^{\cos 3x})$

$\frac{6}{6x}$

8) $\frac{d}{dx}((4\tan(x^2))(\underline{\ln(6x)}))$

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1) $6x - 2$

2) $5\sqrt{x-6} + \frac{5x}{2\sqrt{x-6}}$ *or* $\frac{15x-60}{2\sqrt{x-6}}$

3) $7(5x^3 - 4x)^6(15x^2 - 4)$

4) $\frac{4(2x^2+3) - 4x(4x)}{(2x^2+3)^2} = \frac{12 - 8x^2}{(2x^2+3)^2}$

5) $8\cos 8x$

6) $12e^{4x}$

7) $5^{\cos 3x}(-3\sin 3x)(\ln 5)$

8) $(8x\sec^2 x^2)(\ln 6x) + \frac{4\tan x^2}{x}$

13.2a Derivatives of Vector-Valued Functions

LEARNING TARGETS
At the end of this lesson, you will be able to:

- differentiate a vector-valued function

To differentiate a vector-valued function, simply take the derivative of each component separately.

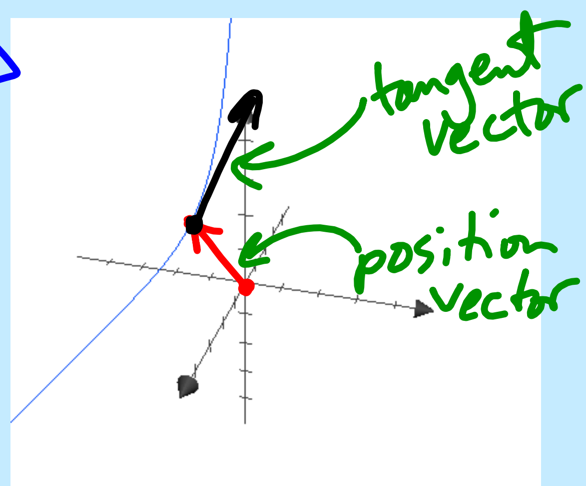
The derivative, $r'(t)$, is called the **tangent vector** and gives the direction of a vector tangent to the curve at any given value of t.

$\langle 1, 0, e^2 \rangle$

ex) $r(t) = \langle 1/t, \ln t, e^{2t} \rangle$ find $r'(t)$. Then sketch the position vector and the tangent vector at t = 1.

$r'(t) = \left\langle \frac{-1}{t^2}, \frac{1}{t}, 2e^{2t} \right\rangle$

$r'(1) = \langle -1, 1, 2e^2 \rangle$



$r'(t) = \langle -1/t^2, 1/t, 2e^{2t} \rangle$

$r'(1) = \langle -1, 1, 2e^2 \rangle$

Smoothness: a curve, r(t), is smooth at all intervals where **r'(t) is continuous and r'(t) ≠ <0, 0> (or <0, 0, 0>**)

Note: all components must equal 0 at the same value of t for a sharp turn (called a 'cusp' or a 'node') to occur

ex) Determine the interval(s) on which
$r(t) = \langle 3t, t^2 - 1, (1/4)t \rangle$ is smooth

✓ $r'(t) = \langle 3, 2t, 1/4 \rangle$ so r(t) is smooth on $(-\infty, \infty)$

$r'(t) = \langle 3, 2t, \frac{1}{4} \rangle$

r(t) is smooth for t in $(-\infty, \infty)$

If r(t) is a <u>smooth</u> curve, then the **unit tangent vector**, T(t) is:

$$T(t) = \frac{r'(t)}{||r'(t)||}$$

The unit tangent vector is a unit vector whose direction is the same as the direction of the curve at any given value of t

ex) Find the unit vector, T(t) tangent to the curve r(t) = $\langle t, t^2 \rangle$. Then find T(1).

$$r'(t) = \langle 1, 2t \rangle$$

$$T(t) = \frac{\langle 1, 2t \rangle}{\sqrt{1+4t^2}} \qquad T(1) = \frac{\langle 1, 2 \rangle}{\sqrt{5}}$$

✓ $T(t) = \frac{\langle 1, 2t \rangle}{\sqrt{1+4t^2}}$ so $T(1) = \frac{\langle 1, 2 \rangle}{\sqrt{5}}$

Practice!

1) Find the interval(s) on $[0, 2\pi]$ on which the vector-valued function $r(\theta) = \langle\theta - 2\cos\theta, \sqrt{3}\theta + 2\sin\theta\rangle$ is smooth

2) Given $r(t) = \langle e^{-t}, t^2, \tan t\rangle$, find: $r'(t) = \langle -e^{-t}, 2t, \sec^2 t\rangle$

$r''(t) = \langle e^{-t}, 2, 2\sec^2 t \tan t\rangle$

a) $r'(t) \cdot r''(t) = -e^{-2t} + 4t + 2\sec^4 t \tan t$

b) $r'(t) \times r(t)$

3) a) Find the angle, θ, between $r(t)$ and $r'(t)$ as a function of t if $r(t) = \langle t^2, t\rangle$.

b) Find all values of t for which $r(t)$ and $r'(t)$ are orthogonal

3a) $r'(t) = \langle 2t, 1\rangle$

$$\cos\theta = \frac{r(t)\cdot r'(t)}{|r(t)||r'(t)|} = \frac{2t^3 + t}{\sqrt{t^4+t^2}\sqrt{4t^2+1}}$$

$$\theta = \arccos\left(\frac{2t^3+t}{\sqrt{t^4+t^2}\sqrt{4t^2+1}}\right)$$

$r(t)\cdot r'(t) = 2t^3 + t = 0$

$t(2t^2+1) = 0$

$t = 0$

$\langle 0, 0\rangle$

$\langle 0, 1\rangle$

1) $r'(\theta) = \langle 1 + 2\sin\theta, \sqrt{3} + 2\cos\theta\rangle$

$\sin\theta = -1/2$ at $\theta = \ldots, 7\pi/6, 11\pi/6, \ldots$

$\cos\theta = -\sqrt{3}/2$ at $\theta = 5\pi/6, 7\pi/6, \ldots$

so $r'(\theta) = \langle 0, 0\rangle$ at $\theta = 7\pi/6$

The intervals of smoothness would be written as

$(0, 7\pi/6)\cup(7\pi/6, 2\pi)$

2) $r'(t) = \langle -e^{-t}, 2t, \sec^2 t\rangle$

a) $-e^{-2t} + 4t + 2\sec^4 t \tan t$

b) $\boxed{\langle 2t\tan t - t^2\sec^2 t, e^{-t}\tan t + e^{-t}\sec^2 t, -t^2e^{-t} - 2te^{-t}\rangle}$

3) $r(t) = \langle t^2, t\rangle$, $r'(t) = \langle 2t, 1\rangle$

a) $\theta = \arccos((2t^3 + t) / \sqrt{(t^4 + t^2)}\sqrt{(4t^2 + 1)})$

$= \arccos((2t^2 + 1) / \sqrt{(t^2 + 1)}\sqrt{(4t^2 + 1)})$

b) for r and r ' to be orthogonal, $2t^3 + t = 0$, so $t = 0$; this leads to the vectors $\langle 0, 0\rangle$ and $\langle 0, 1\rangle$ which seems weird since $\langle 0, 0\rangle$ is more of a point than a vector, so I looked it up and the zero vector is considered orthogonal to every vector (however, you cannot use this for most applications where you need a normal vector).

What have we learned??

- Can I find the derivative of a vector-valued function?