Warmup!

1) Find the distance from point Q(-2, 5, 1) to the plane 2x - y + 3z = 12

2) Find the distance from the point (4, 1, -2) to the line $\frac{x-2}{3} = \frac{y}{2} = z + 1$

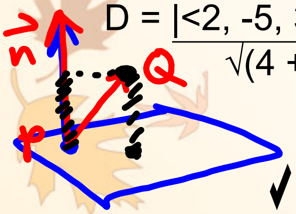
3) Where do the above line and plane intersect?

✓ 1) point P on plane: (0, 0, 4)

vector PQ = <2, -5, 3>

vector n = <2, -1, 3>

$$D = \frac{|\langle 2, -5, 3\rangle \cdot \langle 2, -1, 3\rangle|}{\sqrt{4 + 1 + 9}} = \frac{18}{\sqrt{14}}$$



✓ 2) point P on line: (2, 0, -1)

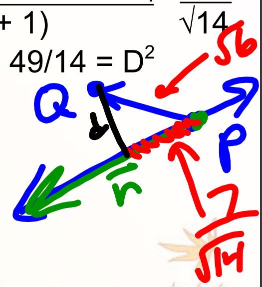
vector PQ = <-2, -1, 1>

vector n = <3, 2, 1>

$$\frac{|\langle -2, -1, 1\rangle \cdot \langle 3, 2, 1\rangle|}{\sqrt{9 + 4 + 1}} = \frac{7}{\sqrt{14}}$$

pythagorus: $6 - 49/14 = D^2$

$D = \sqrt{35/14}$



3) Rewriting the line in parametric form we get:

x = 2 + 3t, y = 2t, z = -1 + t

The plane equation is 2x - y + 3z = 12

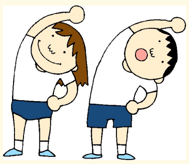
Substituting we get: 2(2 + 3t) - 2t + 3(-1 + t) = 12

so 4 + 6t - 2t - 3 + 3t = 12 -> 7t = 11 -> t = 11/7

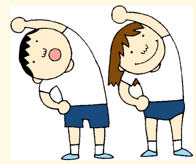
So the point is: x = 2 + 33/7, y = 22/7, z = -1 + 11/7
or (47/7, 22/7, 4/7)

distance from pt to line

$$\frac{|\vec{PQ} \times \vec{u}|}{|\vec{u}|} = \frac{\left\| \begin{matrix} i & j & k \\ -2 & -1 & 1 \\ 3 & 2 & 1 \end{matrix} \right\|}{\sqrt{14}} = \frac{|\langle -3, 5, 1\rangle|}{\sqrt{14}} = \frac{\sqrt{9+25+1}}{\sqrt{14}} = \frac{\sqrt{35}}{\sqrt{14}}$$



WARM UP!!



In your mac search box, type 'grapher'.

Select 3d and click 'choose'

Graph each of the following on your laptop.

$z = y^2$ $z = \sin x$

Ready to have your mind blown?

These are both cylinders!!!

12.6 Surfaces in Space

ESSENTIAL LEARNING TARGETS
At the end of this lesson, you will be able to:

- recognize and write equations for cylindrical surfaces
- recognize and write equations for quadric surfaces

Definition:

If C is a curve in a plane and L is a line, then a **cylinder** is the set of all lines intersecting C that are parallel to each other but not parallel to the plane.

The equation of a cylinder is just the equation of the generating curve, C, with the understanding that in 3-space the rulings (parallel lines) will all run parallel to the missing variable (x, y, or z). So a cylindrical equation will only have 2 variables.

ex) $y^2 + z = 4$ is a cylinder. The generating curve is a parabola and the rulings will be parallel to the x-axis.

September 17, 2019

Quadric Surfaces

Any surface whose equation fits the format:

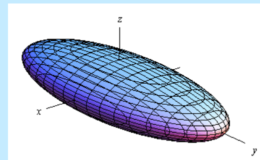
$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$

Note, for this class, the D, E, and F will always be 0. :)

The most common are the following:

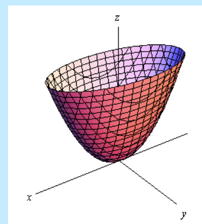
1) Ellipsoid
(This is a sphere if a = b = c ≠ 0)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



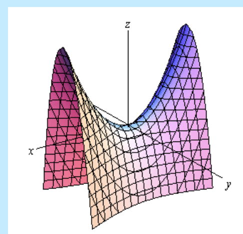
2) Elliptic Paraboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = z$$

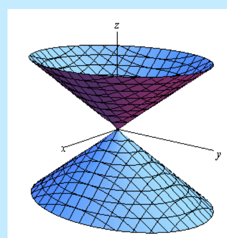


3) Hyperbolic Paraboloid

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}$$

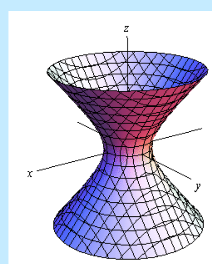


4) Elliptic Cone $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$



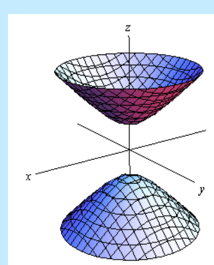
5) Hyperboloid of one sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



6) Hyperboloid of two sheets

$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



ex) Graph each of the following and identify the type of surface

1) $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$	ellipsoid
2) $y^2 + z^2 = 1$	cylinder
3) $4x^2 - y^2 + 2z^2 + 4 = 0$	hyperboloid of one sheet
4) $z = y^2 - x^2$	hyperbolic paraboloid
5) $x^2 + 2z^2 - 6x - y + 10 = 0$	paraboloid

Surfaces of Revolution

In ch 7 we found the area of the surface created by revolving an equation about an axis. Now we are going to find the actual equation for the shape in 3-space.

Remember that the cross section of a surface created by rotation is a circle. So the basic equation we start with is $x^2 + y^2 = r^2$. However, when the radius is not constant, we use a function to represent the changing radius and get $x^2 + y^2 = [r(z)]^2$ (the variables can be switched around depending on the orientation of the graph).

So r(z) is considered the generating curve and $x^2 + y^2$ would be the circular cross sections that follow along the curve up the z-axis.

ex) Write the equation of the surface generated by the curve $z = \ln y$ about the z-axis

Since we are revolving about the z-axis, the circles will be in terms of x and y, so we get $x^2 + y^2 = [r(z)]^2$. In order to get r(z), we need to solve our generating curve for y in terms of z, so we get $y = e^z$.

Our final equation would be $x^2 + y^2 = e^{2z}$

You try! Write the equation for the surface generated by revolving $2z = \sqrt{4 - x^2}$ about the x-axis.

$z = \frac{1}{2}\sqrt{4-x^2}$

$y^2 + z^2 = [r(x)]^2$

$z = 1/2 \sqrt{4 - x^2}$

So the equation would be $y^2 + z^2 = 1/4(4 - x^2)$

about x-axis

What have we learned??

- Can I recognize equations of surfaces in 3-space?