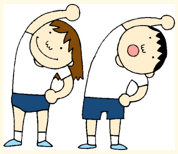
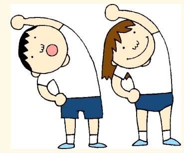


WARM UP!!



1) Find the parametric and symmetric equations of the line that passes through the points (-2, 3, 1) and (4, -8, 3)

2) Find the parametric and symmetric equations of the line that is parallel to the line x = 2 - 3t, y = 4 + t, z = -5 + 4t and passes through the point (1, -4, 7)

3) Find the equation of the plane that contains the points (-1, 3, 6), (0, -2, -5) and (3, 1, -3)

4) Find the equation of the plane that contains the points (4, -2, 1) and (3, 1, -5) and is perpendicular to the plane 3x - 2y + 8z = 5

5) Find the equation of the plane that contains the line x = 1 - 3t, y = 2 + t, z = -4t and makes a 30˚ angle with the positive x-axis

1) direction vector: < 6, -11, 2 >

So the equations would be:

$x = -2 + 6t, y = 3 - 11t, z = 1 + 2t$

Or $\frac{x+2}{\ } = \frac{y-3}{\ } = \frac{z-1}{\ }$

2) direction vector: < -3, 1, 4 >

So the equations would be:

$x = 1 - 3t, y = -4 + t, z = 7 + 4t$

or $\frac{x-1}{\ } = \frac{y+4}{1} = \frac{z-7}{4}$

3) vectors in the plane: < 1, -5, -11 > and < 4, -2, -9 >

$$\begin{vmatrix} i & j & k \\ 1 & -5 & -11 \\ 4 & -2 & -9 \end{vmatrix} = (45 - 22)i - (-9 + 44)j + (-2 + 20)k$$

$= 23i - 35j + 18k$

$23(x + 1) - 35(y - 3) + 18(z - 6) = 0$

$23x - 35y + 18z = -23 - 105 + 108$

$23x - 35y + 18z = -20$

4) vectors in the plane: < -1, 3, -6 > and < 3, -2, 8 >

$$\begin{vmatrix} i & j & k \\ -1 & 3 & -6 \\ 3 & -2 & 8 \end{vmatrix} = (24 - 12)i - (-8 + 18)j + (2 - 9)k$$

$= 12i - 10j - 7k$

$12(x - 4) - 10(y + 2) - 7(z - 1) = 0$

$12x - 10y - 7z = 48 + 20 - 7$

$12x - 10y - 7z = 61$

5) perpendicular vector: < 0, 1, 0 >

$0(x - 6) + 1(y + 1) + 0(z - 4) = 0$

$y = -1$

6) point: (1, 2, 0)

vector on the plane: < -3, 1, -4 >

2nd vector on the plane: < cos30°, 0, sin30° > = <

So < $\sqrt{3}$, 0, 1 > would be on the plane

$$\begin{vmatrix} i & j & k \\ -3 & 1 & -4 \\ \sqrt{3} & 0 & 1 \end{vmatrix} = (1 - 0)i - (-3 + 4\sqrt{3})j + (0 -$$

$1(x - 1) + (3 - 4\sqrt{3})(y - 2) - \sqrt{3}(z - 0) = 0$

$x + (3 - 4\sqrt{3})y - \sqrt{3}z = 1 + 6 - 8\sqrt{3}$

$x + (3 - 4\sqrt{3})y - \sqrt{3}z = 7 - 8\sqrt{3}$

12.5c Lines and Planes in Space!!

ESSENTIAL LEARNING TARGETS
At the end of this lesson, you will be able to:

- find the distances between points, lines and planes in space

We know how to find the distance between 2 points, but can you find the distance between a point and a plane, a point and a line, or a plane and another plane?

Let's see if you can figure it out. Find the distance between the point Q(1, 5, -4) and the plane 3x - y + 2z = 6.

Hint:

find a ... about project

$P = (2, 0, 0)$

$\vec{PQ} = \langle -1, 5, -4 \rangle$

$\vec{n} = \langle 3, -1, 2 \rangle$

$$\text{Proj}_n \vec{PQ} = \frac{\vec{PQ} \cdot \vec{n}}{\|\vec{n}\|} = \left| \frac{-3-5-8}{\sqrt{9+1+4}} \right| = \frac{16}{\sqrt{14}}$$

$n = \langle 3, -1, 2 \rangle$ is a vector perpendicular to the plane.
A point on the plane would be P(0, 0, 3) so a
Vector from P(0, 0, 3) to Q(1, 5, -4) would be $\overrightarrow{PQ} = \langle 1, 5, -7 \rangle$.
We want the length of the projection of PQ onto n.
If the projection of PQ onto n is $\frac{PQ \cdot n}{||n||^2}$(n) and $\frac{n}{||n||}$ is a unit vector,
The length of the projection of PQ onto n is $\frac{|PQ \cdot n|}{||n||}$

$$= \frac{|\langle 1, 5, -7 \rangle \cdot \langle 3, -1, 2 \rangle|}{\sqrt{3^2 + (-1)^2 + 2^2}} = \frac{|3 - 5 - 14|}{\sqrt{14}} = \frac{16}{\sqrt{14}}$$

So in general, to find the distance from a point, Q, to a plane:

- find a vector, n, perpendicular to the plane
- find a point, P, on the plane
- find vector PQ
- find the length of the projection of PQ onto n

OR the Noah Feist method:

- find a vector, n, perpendicular to the plane
- find the equation of the line in the direction of n through Q
- find the point of intersection of the line and the plane
- find the distance between the point of intersection and Q

Formula!

If you just want to memorize a formula, you can memorize the following:

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Where x_1, y_1, z_1 is the point and a, b, c, d are from the equation of the plane written in standard form and set = 0.

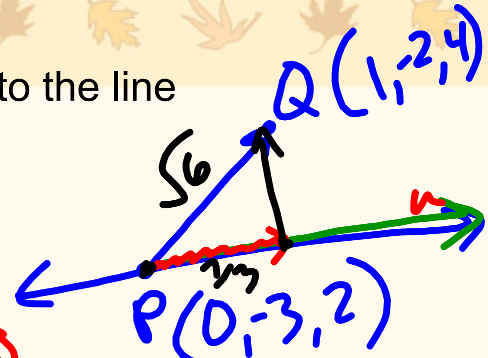
$3x - y + 2z = 6 \rightarrow 3x - y + 2z - 6 = 0$ (a, b, c, d)

$(1, 5, -4)$

$$D = \frac{|3(1) + -1(5) + 2(-4) + -6|}{\sqrt{9 + 1 + 4}} = \frac{|-16|}{\sqrt{14}} = \frac{16}{\sqrt{14}}$$

Can you figure out another one?

Find the distance from the point Q(1, -2, 4) to the line
x = 2t, y = t - 3, z = 2t + 2



direction vector for the line: u = <2, 1, 2>

point on the line (letting t = 0): P(0, -3, 2)

vector PQ = <-1, -1, -2>

$$||proj_u PQ|| = \frac{|\langle 2, 1, 2\rangle \cdot \langle -1, -1, -2\rangle|}{\sqrt{(4 + 1 + 4)}} = \frac{|-2 - 1 - 4|}{3} = 7/3$$

$$\left(\frac{7}{3}\right)^2 + d^2 = 6$$

$||PQ|| = \sqrt{(1 + 1 + 4)} = \sqrt{6}$

by pythagorus: distance = $\sqrt{(6 - 49/9)} = \sqrt{5}/3$

or you could subtract the projection from PQ to find the vector projection orthogonal to the direction vector and then find the magnitude of that vector

So to find the distance between a point, Q, and a line:

- find a point, P, on the line
- find vector PQ
- find a direction vector, u, for the line
- find the length of the projection of PQ onto u
- use pythagorus to find the distance
  - > or subtract the projection from PQ to find the orthogonal vector and find its magnitude

OR there is a formula!

- find a point, P, on the line
- find vector PQ
- find a direction vector, u, for the line
  - > $D = \frac{||PQ \times u||}{||u||}$

per Josie Schmitt, the magnitude of PQ x u would be the area of a rectangle where PQ is the diagonal and u is a side, so when you divide out the magnitude of u you are left with the other side

Last one: plane and a plane

(2, 0, 0)

Find the distance between 3x - y + 2z = 6 and 6x - 2y + 4z = -4. (Note the planes have to be parallel or this would just be silly.)

Hint: this is very similar to the distance between a point and a plane

Point on first plane: Q(0, 0, 3)

Point on second plane: P(0, 0, -1)

vector PQ: < 0, 0, -4 >

vector perpendicular to second plane: n = < 6, -2, 4 >

length of projection of PQ onto n: $\dfrac{|\langle 0, 0, -4\rangle \cdot \langle 6, -2, 4\rangle|}{\sqrt{(36 + 4 + 16)}} = \dfrac{16}{\sqrt{56}} = \dfrac{16}{2\sqrt{14}}$

Find the line of intersection of the planes:

x + y + z = 1 and x - 2y + 3z = 1

$\vec{u} = \langle 1, 1, 1 \rangle$ $\quad$ $\vec{v} = \langle 1, -2, 3 \rangle$

$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = \langle 5, -2, -3 \rangle$ dir vector

Set $z = 0$

$x - 2y = 1$

$x + y = 1$

$-3y = 0$

$y = 0$

$x = 1$

pt: $(1, 0, 0)$

$x = 1 + 5t$

$y = -2t$

$z = -3t$

Is there time to review? Of course!

1) Find the distance from point Q(-2, 5, 1) to the plane 2x - y + 3z = 12

2) Find the distance from the point (4, 1, -2) to the line $\frac{x-2}{3} = \frac{y}{2} = z + 1$

3) Where do the above line and plane intersect?

1) point P on plane: (0, 0, 4)

vector PQ = <2, -5, 3>

vector n = <2, -1, 3>

$$D = \frac{|\langle 2, -5, 3\rangle \bullet \langle 2, -1, 3\rangle|}{\sqrt{(4 + 1 + 9)}} = \frac{18}{\sqrt{14}}$$

2) point P on line: (2, 0, -1)

vector PQ = <-2, -1, 1>

vector n = <3, 2, 1>

$$\frac{|\langle -2, -1, 1\rangle \bullet \langle 3, 2, 1\rangle|}{\sqrt{(9 + 4 + 1)}} = \frac{7}{\sqrt{14}}$$

pythagorus: $6 - 49/14 = D^2$

$D = \sqrt{(35/14)}$

3) Rewriting the line in parametric form we get:

x = 2 + 3t, y = 2t, z = -1 + t

The plane equation is 2x - y + 3z = 12

Substituting we get: 2(2 + 3t) - 2t + 3(-1 + t) = 12

so 4 + 6t - 2t - 3 + 3t = 12 -> 7t = 11 -> t = 11/7

So the point is: x = 2 + 33/7, y = 22/7, z = -1 + 11/7
or (47/7, 22/7, 4/7)

What have we learned??

- Can I find the distance between points, lines and planes in space?