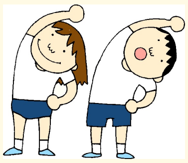
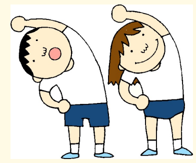


WARM UP!!



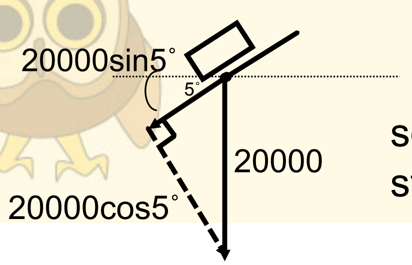
1) Find 2 nonzero vectors orthogonal to $\langle -3, 5, -2 \rangle$

2) A 20000 pound truck sits on a 5° incline slope. Assuming no friction, what is the force necessary to keep the truck from rolling down the hill?

3) An object is pulled 20 feet across a floor with a force of 112 pounds from a direction of 50°. Find the work done.

✓ 1) Any vectors where the dot product is zero.
My two are: $\langle 1, 1, 1 \rangle$ and $\langle -1, -1, -1 \rangle$

✓ 2)



so the force required to keep the truck stationary is $20000\sin 5^\circ \approx 1743.115$ pounds

✓ 3) distance vector: $\langle 20, 0 \rangle$

force vector: $\langle 112\cos 50^\circ, 112\sin 50^\circ \rangle$

$W = \langle 20, 0 \rangle \cdot \langle 112\cos 50^\circ, 112\sin 50^\circ \rangle$

$w = (112\cos 50^\circ)(20) + 0 \approx 1439.844$ foot-pounds

12.5b Planes in Space!!

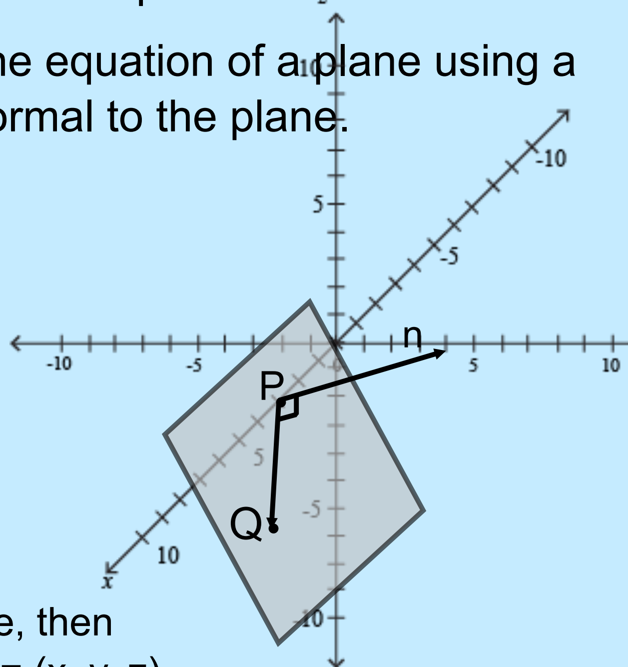
ESSENTIAL LEARNING TARGETS
At the end of this lesson, you will be able to:

- write a linear equation to represent a plane in space
- find the angle between planes in space

Yesterday, we learned how to write the equation of a line given a point on the line and a direction vector parallel to the line.

Today we will learn how to write the equation of a plane using a point on the plane and a vector normal to the plane.

Think about it: If P is a point on the plane, and n is a vector perpendicular to the plane, then the plane consists of all vectors through P that are orthogonal to n.



So if Q is any other point on the plane, then n • PQ = 0. If we let n = <a, b, c>, Q = (x, y, z) and P = (x_1, y_1, z_1), then we get:

$\langle a, b, c\rangle \cdot \langle x - x_1, y - y_1, z - z_1\rangle = 0$

OR $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

So, if a plane contains point (x_1, y_1, z_1) and vector <a, b, c> is normal to the plane, then the equation of the plane is:

Standard form: $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

General form: $ax + by + cz + d = 0$

ex) Find the equation of the plane that contains points (2, 1, 1), (0, 4, 1) and (-2, 1, 4)

Since we don't have a normal vector, create 2 vectors using the given points and then find their cross product to get an orthogonal vector.

Let u = <-2, 3, 0> and v = <-4, 0, 3>

$$u \times v = \begin{bmatrix} i & j & k \\ -2 & 3 & 0 \\ -4 & 0 & 3 \end{bmatrix} \begin{matrix} i & j \\ -2 & 3 \\ -4 & 0 \end{matrix} = 9i + 0j + 0k + 12k + 0i + 6j$$

$$= 9i + 6j + 12k = \langle 9, 6, 12 \rangle$$

$n = \langle 3, 2, 4 \rangle$

So the equation of the plane is

$9(x - 2) + 6(y - 1) + 12(z - 1) = 0$ OR

$9x + 6y + 12z - 36 = 0$ OR

$3x + 2y + 4z - 12 = 0$

You try! Find the equation of the plane that passes through (1, 2, 3) and is parallel to the yz-plane.

A vector normal to the plane would be <1, 0, 0>
so an equation of the plane would be:

$1(x - 1) + 0(y - 2) + 0(z - 3) = 0$ OR $x = 1$

$20x - 20 = 0$

$x = 1$

You try again! Find the equation of the plane that passes through the points (3, 2, 1) and (3, 1, -5) and is perpendicular to the plane 6x + 7y + 2z = 10.

A vector on the plane would be <0, -1, -6>.

The vector normal to the perpendicular plane would be <6, 7, 2>. But normal to perpendicular means that it would be parallel to the original plane. So use the cross product to get an orthogonal vector.

$$u \times v = \begin{bmatrix} i & j & k \\ 0 & -1 & -6 \\ 6 & 7 & 2 \end{bmatrix} \begin{matrix} i & j \\ 0 & -1 \\ 6 & 7 \end{matrix} = -2i - 36j + 0k + 6k + 42i + 0j$$

$$= 40i - 36j + 6k$$

So the equation would be:
40(x - 3) - 36(y - 2) + 6(z - 1) = 0 OR
40x - 36y + 6z - 54 = 0 OR
20x - 18y + 3z - 27 = 0

To find the angle between 2 planes, you need to use the vectors normal to the planes.

If n_1 and n_2 are normal to 2 planes,
and θ is the angle between the planes, then

$$\cos\theta = \frac{|n_1 \cdot n_2|}{||n_1||\,||n_2||}$$

If the planes are parallel, then $n_1 = kn_2$
(they are scalar multiples)

If the planes are perpendicular, then $n_1 \cdot n_2 = 0$

ex) Determine whether the planes below are parallel, orthogonal, or if neither, find the angle between them.

a) $3x + y - 4z = 3$ and $-9x - 3y + 12z = 4$

$n_1 = \langle 3, 1, -4 \rangle$ and $n_2 = \langle -9, -3, 12 \rangle$

Since $-3n_1 = n_2$, these planes are parallel

b) $3x + 2y - z = 7$ and $x - 4y + 2z = 0$

$n_1 = \langle 3, 2, -1 \rangle$ and $n_2 = \langle 1, -4, 2 \rangle$

$n_1 \cdot n_2 = 3 - 8 - 2 = -7$

$\cos\theta = |-7| / (\sqrt{(9 + 4 + 1)}\sqrt{(1 + 16 + 4)})$

$= 7 / (\sqrt{14}\sqrt{21})$

$\theta \approx 65.91^\circ$

Let's do some review!

Suppose u = <2, -1, -5>, v = <-4, 2, -1> and w = <1, 6, 2>. Find the following:

a) u • v
b) u x v
c) u x w
d) w x u
e) the angle between u and v
f) the angle between v and w
g) the area of the triangle with sides u and w
h) the area of the parallelogram with adjacent sides u and v
i) the projection of w onto v

$a)\ u\cdot v = 2(-4) + -1(2) + -5(-1) = -5$

$$b)\ u\times v = \begin{vmatrix} i & j & k \\ 2 & -1 & -5 \\ -4 & 2 & -1 \end{vmatrix} \begin{matrix} i & j \\ 2 & -1 \\ -4 & 2 \end{matrix} = i + 20j + 4k - 4k + 10i + 2j = \langle 11, 22, 0\rangle$$

$$c)\ u\times w = \begin{vmatrix} i & j & k \\ 2 & -1 & -5 \\ 1 & 6 & 2 \end{vmatrix} \begin{matrix} i & j \\ 2 & -1 \\ 1 & 6 \end{matrix} = -2i - 5j + 12k + k + 30i - 4j = \langle 28, -9, 13\rangle$$

$d)\ u\times w = -(w\times u) = \langle -28, 9, -13\rangle$

$$e)\ \cos\theta = \frac{u\cdot v}{||u||\,||v||} = \frac{-5}{\sqrt{4+1+25}\sqrt{16+4+1}} = \frac{-5}{\sqrt{30}\sqrt{21}}\ so\ \ \theta\approx 101.49^\circ$$

$$f)\ \cos\theta = \frac{v\cdot w}{||v||\,||w||} = \frac{-4+12-2}{\sqrt{16+4+1}\sqrt{1+36+4}} = \frac{6}{\sqrt{21}\sqrt{41}}\ so\ \ \theta\approx 78.2^\circ$$

$$g)\ \frac{1}{2}\|\langle 28, -9, 13\rangle\| \approx 16.078$$

$h)\ \|\langle 11, 22, 0\rangle\| \approx 24.597$

$$i)\ proj_v w = \left(\frac{w\cdot v}{||v||^2}\right)v = \left(\frac{-4+12-2}{16+4+1}\right)\langle -4, 2, -1\rangle = \frac{6}{21}\langle -4, 2, -1\rangle\ or\ \frac{2}{7}\langle -4, 2, -1\rangle$$

What have we learned??

- Can I write a linear equation to represent a plane in space?
- Can I find the angle between planes in space?

CORE

HW (math)

quiz (math)

Help

September 12, 2019