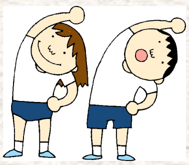
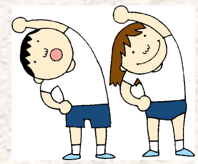


WARM UP!!



A plane flies with a constant ground speed of 425 mph at E52˚N. It is encountering a 64 mph wind from the northwest. Find the airspeed of the plane and the "heading" at which it must be steered to maintain its desired course and speed.

let R = 425<cos52˚, sin52˚> ≈ <261.656, 334.905>

let W = 64<cos -45˚, sin -45˚> ≈ <45.255, -45.255>

we want C + W = R, so C ≈ <216.401, 380.159>

tanθ = 380.159/216.401 so θ ≈ 60.3498

So the plane should fly at 437.437 mph at 60.35˚

(or N29.65˚E)

12.5a Lines in Space!!

LEARNING TARGET
At the end of this lesson, you will be able to:

- write a set of parametric equations for a line in space

Parametric Equations of a Line in Space

A line L parallel to the vector v = <a, b, c> and passing through the point $P(x_1, y_1, z_1)$ is represented by the parametric equations below. (You can think of the direction vector like your slope.)

$x = x_1 + at, \quad y = y_1 + bt, \quad z = z_1 + ct$

Symmetric Equations of a Line in Space

If a, b, and c of vector v are all nonzero, then the symmetric equations of the line are below. (Just solving the parametric equations for t.)

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

ex) Find parametric and symmetric equations of the line L that passes through the point (1, -2, 4) and is parallel to v = <2, 4, -4>

parametric

$x = 1 + 2t, \; y = -2 + 4t, \; z = 4 - 4t$

$x = 1 + t, \; y = -2 + 2t, \; z = 4 - 2t$ ☺

symmetric

$$\frac{x - 1}{2} = \frac{y + 2}{4} = \frac{z - 4}{-4}$$

ex) Find a set of parametric equations of the line that passes through the points (-2, 1, 0) and (1, 3, 5)

✓ v = <3, 2, 5>

parametric equations:

✓ x = -2 + 3t, y = 1 + 2t, z = 5t

Can you do it backwards?

Find the coordinates of a point, P, on the line and a vector, v, parallel to the line, x = 4t, y = 5 - t and z = 4 + 3t

✓ Using t = 0, I got a point of P(0, 5, 4) and a direction vector of <4, -1, 3>

Determine whether the lines intersect, and if they do, find the point of intersection and the cosine of the angle of intersection for:

$$\frac{x-2}{-3} = \frac{y-2}{6} = z - 3 \quad \text{and} \quad \frac{x-3}{2} = y + 5 = \frac{z+2}{4}$$

For the **point of intersection**, write parametric equations:

$x = 2 - 3t$, $y = 2 + 6t$, $z = 3 + t$

$x = 3 + 2q$, $y = -5 + q$, $z = -2 + 4q$

Let any constants help you:

$2 - 3t = 3 + 2q$

$2 + 6t = -5 + q$

$4 - 6t = 6 + 4q$

$6 = 1 + 5q$ so $q = 1$

At $q = 1$, we get the point $(5, -4, 2)$

$2 - 3t = 5$ at $t = -1$

At $t = -1$, we get the point $(5, -4, 2)$

Match! So the point of intersection is: $(5, -4, 2)$

For the **cosine of the angle**:

direction vector for first line is $\langle -3, 6, 1\rangle$

direction vector for second line is $\langle 2, 1, 4\rangle$

$$\cos\theta = \frac{\langle -3, 6, 1\rangle \cdot \langle 2, 1, 4\rangle}{\|\langle -3, 6, 1\rangle\|\|\langle 2, 1, 4\rangle\|} = \frac{-6 + 6 + 4}{\sqrt{46}\,\sqrt{21}} = \frac{4}{\sqrt{966}}$$

Review!

Given points A(2, 1, -6), B(1, 4, -3) and C(-1, 0, 2), find the following:

a) the projection of vector $\overrightarrow{AC}$ onto vector $\overrightarrow{AB}$

b) the angle between $\overrightarrow{BC}$ and $\overrightarrow{AC}$

c) the vector orthogonal to $\overrightarrow{BC}$ and $\overrightarrow{BA}$

d) the parametric equations of AB

e) the symmetric equations of AC

$\overrightarrow{AB} = \langle -1, 3, 3\rangle$, $\overrightarrow{AC} = \langle -3, -1, 8\rangle$, $\overrightarrow{BC} = \langle -2, -4, 5\rangle$, $\overrightarrow{BA} = \langle 1, -3, -3\rangle$

$$proj_{\overrightarrow{AB}}\overrightarrow{AC} = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{\left\|\overrightarrow{AB}\right\|^2}\overrightarrow{AB} = \frac{3-3+24}{1+9+9}\langle -1, 3, 3\rangle = \langle -\frac{24}{19}, \frac{72}{19}, \frac{72}{19}\rangle$$

$$\cos\theta = \frac{\overrightarrow{AC}\cdot\overrightarrow{BC}}{||AC||\,||BC||} = \frac{6+4+40}{\sqrt{9+1+64}\sqrt{4+16+25}} = \frac{50}{\sqrt{74}\sqrt{45}}$$

$\theta \approx 29.95°$

$$\overrightarrow{BC}\times\overrightarrow{BA} = \begin{vmatrix} i & j & k \\ -2 & -4 & 5 \\ 1 & -3 & -3 \end{vmatrix}\begin{matrix} i & j \\ -2 & -4 \\ 1 & -3 \end{matrix} = 12i + 5j + 6k + 4k + 15i - 6j$$

$\langle 27, -1, 10\rangle$

Parametric equations of $\overrightarrow{AB}$: $x = 2 - t, y = 1 + 3t, z = -6 + 3t$

Symmetric equations of $\overrightarrow{AC}$: $\frac{x-2}{-3} = \frac{y-1}{-1} = \frac{z+6}{8}$

What have we learned??

- Can I write a set of parametric equations for a line in space?