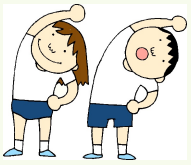
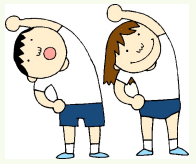


WARM UP!!

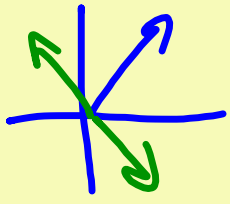


Complete the following table:

	Vector	|| ||	angle with x-axis in degrees (round to hundredth)	unit vector in same direction
1)	$\langle 2, 3 \rangle$	$\sqrt{13}$	56.31°	$\langle 2, 3 \rangle / \sqrt{13}$
2)	$\langle \cos 37°, \sin 37° \rangle$	1	37°	$\langle \cos 37°, \sin 37° \rangle$
3)	$14\langle 1/2, \sqrt{3}/2 \rangle$	14	60°	$\langle 1/2, \sqrt{3}/2 \rangle$
4)	$3\langle 4, 1 \rangle$	$3\sqrt{17}$	14.04°	$\langle 4, 1 \rangle / \sqrt{17}$
5)	$\langle 7, 2 \rangle / \sqrt{53}$	1	15.95°	$\langle 7, 2 \rangle / \sqrt{53}$
6)	$\langle \cos 60°, \sin 30° \rangle$	$1/\sqrt{2}$	45°	$\sqrt{2}\langle 1/2, 1/2 \rangle$
7)	$4i + 2j$	$2\sqrt{5}$	26.57°	$\langle 4, 2 \rangle / 2\sqrt{5}$
8)	$5i$	5	0°	$\langle 1, 0 \rangle$
9)	$\langle -5, 1 \rangle$	$\sqrt{26}$	168.69°	$\langle -5, 1 \rangle / \sqrt{26}$
10)	$3i - j$	$\sqrt{10}$	-18.43° or 341.57°	$\langle 3, -1 \rangle / \sqrt{10}$

Quick warm-up follow-up question:

Suppose I started with the vector $\langle 3, 4 \rangle$. How could I write a unit vector normal to the original vector?



unit vector in the same direction: $\langle 3, 4 \rangle / 5$

unit vector normal to original: $\pm \langle -4, 3 \rangle / 5$

(Why is the ± necessary?)

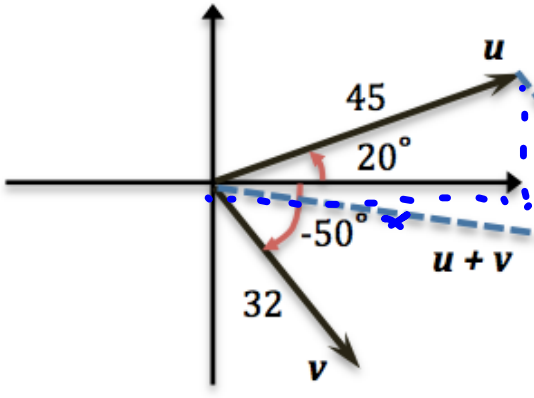
12.2b Resultant Forces!!

ESSENTIAL LEARNING TARGETS

At the end of this lesson, you will be able to:

- use vectors to solve problems involving force or velocity

ex) Find the magnitude and direction ***u*** + ***v*** if ***u*** and ***v*** are the vectors below:



$u = \langle 45\cos 20^\circ, 45\sin 20^\circ \rangle$

$v = \langle 32\cos(-50^\circ), 32\sin(-50^\circ) \rangle$

$u + v = \langle 45\cos 20^\circ + 32\cos -50^\circ, 45\sin 20^\circ + 32\sin -50^\circ \rangle$

$\approx \langle 62.8553, -9.1225 \rangle$

$\|u + v\| = \sqrt{(62.8553)^2 + (-9.1225)^2} \approx 63.5139$

$\text{direction} = \arctan\left(\frac{-9.1225}{62.8553}\right) \approx -8.2579^\circ$

If ***u*** and ***v*** are vectors, then ***u*** + ***v*** is called the resultant vector. Geometrically this is the diagonal of a parallelogram having ***u*** and ***v*** as its adjacent sides.

If two or more forces are acting on an object, then the resultant force on the object is the vector sum of the vector forces.

ex) Three forces with magnitudes of 400 newtons, 280 newtons and 350 newtons act on an object at angles of -30˚, 45˚, and 135˚ respectively, with the positive x-axis. Find the direction and magnitude of the resultant force.

$F_1 = 400\langle\cos(-30^\circ), \sin(-30^\circ)\rangle$

$F_2 = 280\langle\cos45^\circ, \sin45^\circ\rangle$

$F_3 = 350\langle\cos135^\circ, \sin135^\circ\rangle$

$F_R = F_1+F_2+F_3 = \langle 400\cos{-30^\circ}+280\cos45^\circ+350\cos135^\circ, 400\sin{-30^\circ}+280\sin45^\circ+350\sin135^\circ\rangle \approx \langle 296.912, 245.477\rangle$

$\| F_R \| \approx 385.248$

direction $\approx \tan^{-1}(245.477/296.912) \approx 39.582^\circ$

An airplane's course is set at N30˚W. Its speed is set at 500 mph. It encounters a wind blowing N45˚E at 70 mph. What is the resultant speed and direction?

A bearing of N30˚W means 30˚ west of north which equates to an angle of 120˚ on the unit circle.

A bearing of N45˚E means 45˚ east of north which equates to an angle of 45˚ on the unit circle.

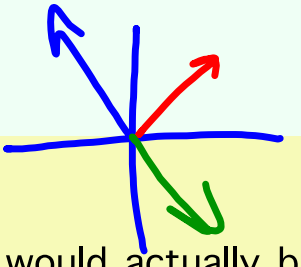
$F_1 = 500\langle \cos 120^\circ, \sin 120^\circ \rangle$

$F_2 = 70\langle \cos 45^\circ, \sin 45^\circ \rangle$

$F_R = \langle 500\cos 120^\circ + 70\cos 45^\circ, 500\sin 120^\circ + 70\sin 45^\circ \rangle$

$\approx \langle -200.503, 482.510 \rangle$

$\| F_R \| \approx 522.511$ mph



direction $\approx -67.435^\circ$

Are you sure?? Be careful! The direction would actually be in the 2nd quadrant (if not sure, just look at the resultant vector), so the angle is actually 112.564˚

An airplane's course is set at N20˚E. Its speed is set at 420 mph. It encounters a wind blowing N80˚W at 92 mph. What is the resultant speed and direction?

A bearing of N20˚E means 20˚ east of north which equates to an angle of 70˚ on the unit circle.

A bearing of N80˚W means 80˚ west of north which equates to an angle of 170˚ on the unit circle.

$F_1 = 420\langle \cos 70^\circ, \sin 70^\circ \rangle$

$F_2 = 92\langle \cos 170^\circ, \sin 170^\circ \rangle$

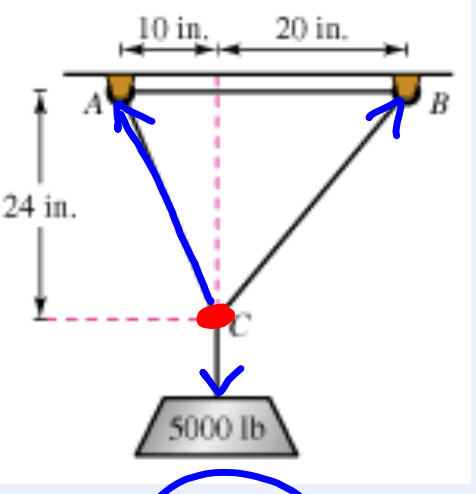
$F_R = \langle 420\cos 70^\circ + 92\cos 170^\circ, 420\sin 70^\circ + 92\sin 170^\circ \rangle$

$\approx \langle 53.046, 410.647 \rangle$

$\| F_R \| \approx 414.058$ mph

direction $\approx 82.639^\circ$

ex) Determine the tension in each cable supporting the given load:



a<-10, 24> + b<20, 24> = 5000<0, 1>

<-10a, 24a> + <20b, 24b> = <0, 5000>

-10a + 20b = 0 -> a = 2b

24a + 24b = 5000 -> 48b + 24b = 5000

so b = 5000/72 = 625/9

a = 1250/9

Tension on AC = || 1250/9 < -10, 24 > || ≈ 3611.111

Tension on BC = || 625/9 < 20, 24 > || ≈ 2169.514

What have we learned??

- What is a resultant vector?
- How does the magnitude of a resultant vector relate to the force on an object?