Quick Reference for PVA with vectors

position vector: $\langle x(t), y(t) \rangle$

velocity vector: $\langle x'(t), y'(t) \rangle$ or $\langle dx/dt,\ dy/dt \rangle$

acceleration vector: $\langle x''(t), y''(t) \rangle$

speed is the magnitude of the velocity vector, so

speed = $\sqrt{(x'(t))^2 + (y'(t))^2}$

distance = $\int |\text{velocity}| = \int \text{speed}$

$$\text{Slope} = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

PVA problems on the BC calculus exam usually are written as parametric equations in vector form. The math doesn't change, just the format of the answers.

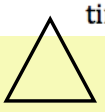
2015 #2) (calculator) At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = (\cos(t^2), e^{0.5t})$. At $t = 1$, the particle is at the point $(3, 5)$.

a) Find the x-coordinate of the position of the particle at time $t = 2$.

b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

c) Find the time at which the speed of the particle is 3.

d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.



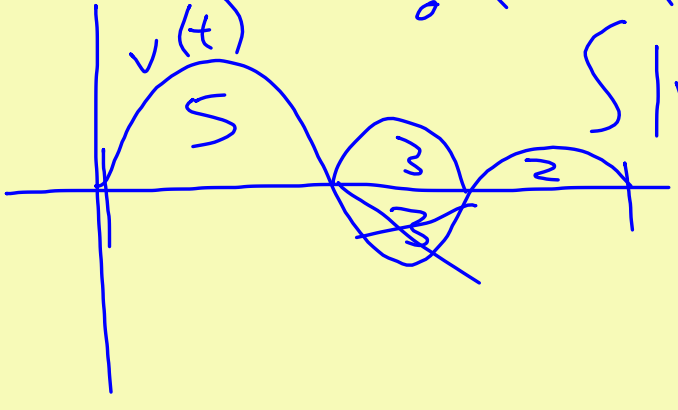
a) $\int_1^2 v_x(t)\,dt = x(2) - x(1)$

$x(2) = 3 + \int_1^2 v_x(t)\,dt \approx 2.557$

b) $\text{slope} = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^{.5t}}{\cos t^2} = 2$

c) $\text{speed} = \sqrt{(\cos(t^2))^2 + (e^{.5t})^2} = 3$

d) $\text{distance} = \int_0^1 \sqrt{(\cos t^2)^2 + (e^{.5t})^2}\,dt$

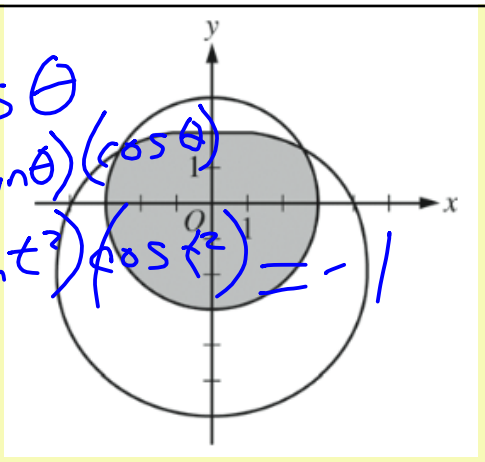


$\int |v(t)|\,dt$

$x(\theta) = r\cos\theta$

$x(\theta) = (4 - 2\sin\theta)(\cos\theta)$

$x(t) = (4 - 2\sin t^2)(\cos t^2) = -1$



2013 #2

The graphs of the polar curves r = 3 and r = 4 - 2sinθ are shown in the figure above. The curves intersect when θ = π/6 and θ = 5π/6.

a) Let S be the shaded region that is inside the graph of r = 3 and also inside the graph of r = 4 - 2sinθ. Find the area of S.

b) A particle moves along the polar curve r = 4 - 2sinθ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time t = 1.5.

$y_1 = (4 - 2\sin x^2)(\cos x^2)$

graph

calc → $\frac{dy}{dx}$ → 1.5 (enter)

$\boxed{89}$

$\frac{d}{dx}\left(x(t), x\right)\Big|_{x=1.5}$

position: $x(t)$

velocity: $v(t) = x'(t) = \frac{dx}{dt}$

acceleration: $a(t) = v'(t) = \frac{d^2x}{dt^2}$

now

pos: $\langle x(t), y(t) \rangle$

vel: $\langle x'(t), y'(t) \rangle = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle$

acc: $\langle x''(t), y''(t) \rangle$

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