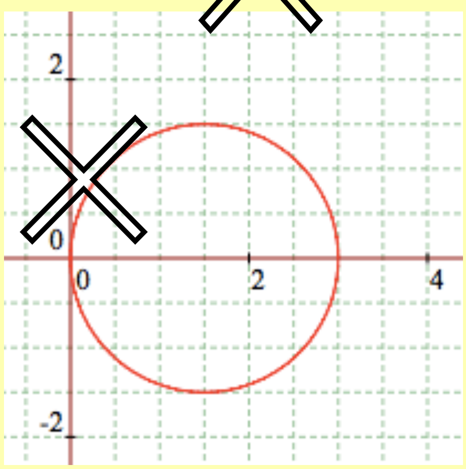
WARMUP!!

Problem #3 on:

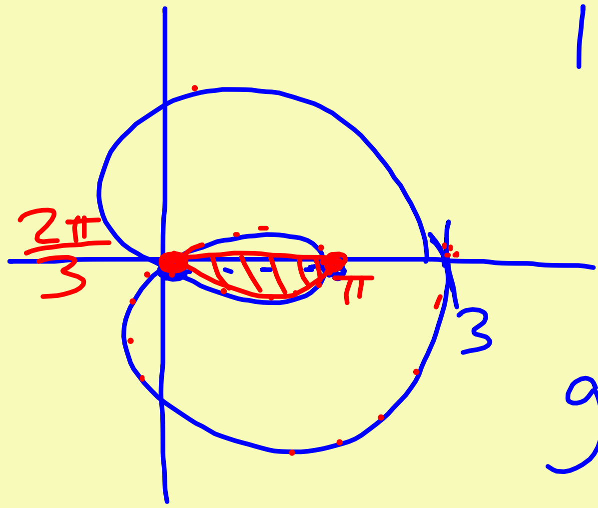
Solution:



Area = $\pi(3/2)^2 = 9/4\ \pi$

(13) area of inner loop of

$r = 1 + 2\cos\theta$



$1 + 2\cos\theta = 0$

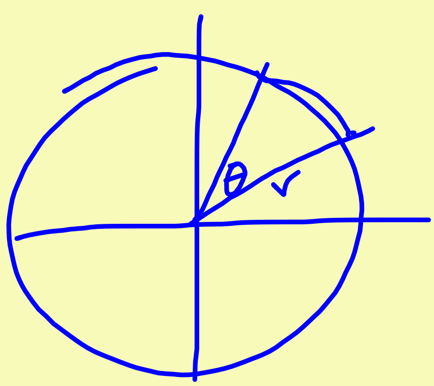
$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

go $\frac{2\pi}{3}$ to π and double

$$2 \cdot \frac{1}{2} \int_{\frac{2\pi}{3}}^{\pi} (1 + 2\cos\theta)^2 \, d\theta$$

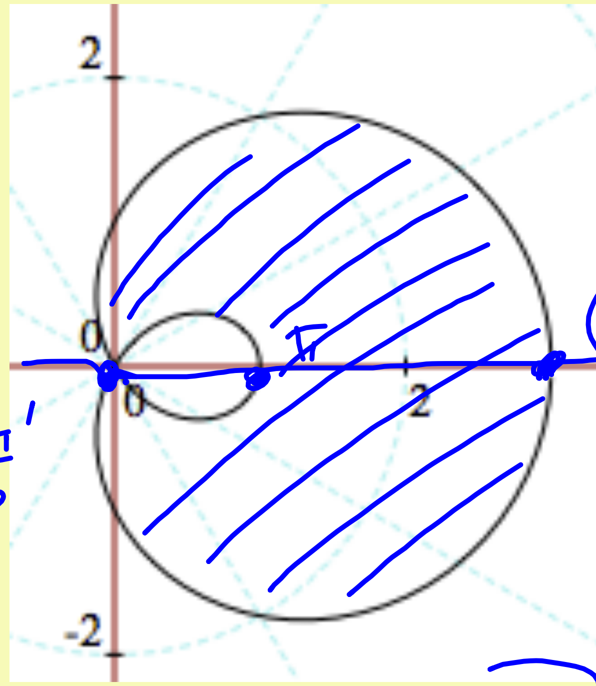
$$A = \frac{1}{2} \theta r^2$$



$$\frac{1}{2} \int_{\alpha}^{\beta} r^2 \, d\theta$$

(15)

$r = 1 + 2\cos\theta$



$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$$A = 2\left[\frac{1}{2}\int_0^{\frac{2\pi}{3}} r^2\,d\theta - \frac{1}{2}\int_{\frac{2\pi}{3}}^{\pi} r^2\,d\theta\right]$$

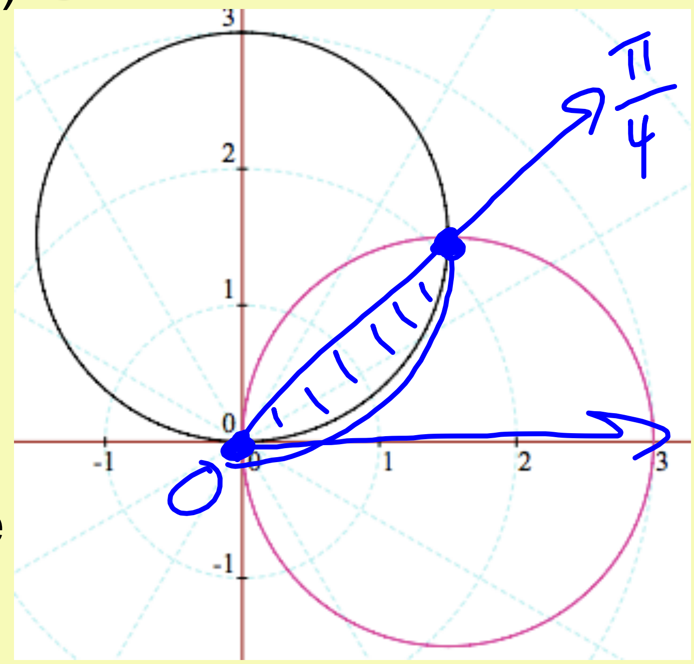
10.5b - Arc Length on Polar Graphs!

At the end of this lesson the students will be able to:

- find the length of a portion of a graph of a polar equation

ex) Find the area of the region in common to the graphs of r = 3cosθ and r = 3sinθ = 0

(Calculators permitted) 😇



3cosθ = 3sinθ

tanθ = 1

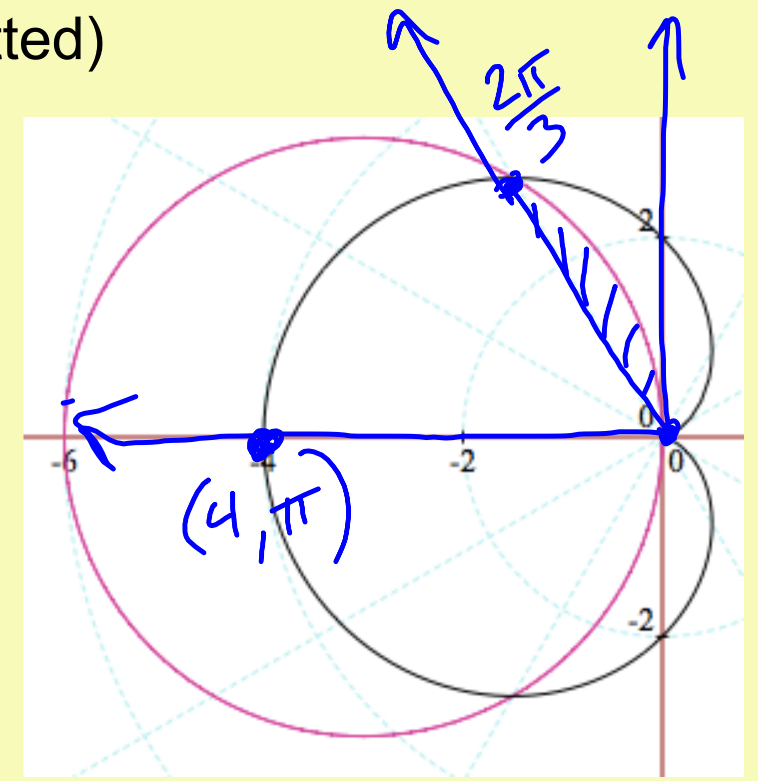
θ = π/4

The area is simply double the area of the region of either circle from 0 to π/4.

$$A = 2\left[\frac{1}{2}\int_0^{\pi/4} (3\sin\theta)^2 d\theta\right] = \frac{9\pi}{8} - \frac{9}{4} \approx 1.284$$

You try) Find the area of the region in common to the graphs of r = -6cosθ and r = 2 - 2cosθ. = 4
(Calculators permitted)

Symmetry says we can just focus on 0 to π and double it, so the point of intersection occurs at θ = 2π/3.



So we can find the area within the cardiod from 2π/3 to π and then add the area of the circle from π/2 to 2π/3.

$$A = 2\left[\frac{1}{2}\int_{\frac{2\pi}{3}}^{\pi}(2-2\cos\theta)^2 d\theta + \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}}(-6\cos\theta)^2 d\theta\right] = 5\pi$$

ARC LENGTH FORMULAS

Arc length of a traditionally defined function: y = f(x)

$$S = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\, dx$$

Arc length of a parametrically defined function

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Arc length of a polar function

$$S = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

ex) Find the length of the curve r = 8(1 + cosθ) on [0, 2π]

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

You can use a calculator to check out the graph but no calculator for evaluating the integral - hee, hee, hee

$2\cos^2\theta = 1+\cos 2\theta$

Hint: it's always good to check out the graph to see if you can use symmetry. Sometimes using limits of 0 and 2π can mess you up because sine and cosine have the same values at those limits and often subtract out to 0.

Hint #2: when in doubt with a root, try rationalizing or applying a reverse power reduction identity

$$S = 2\int_0^{\pi} \sqrt{(8+8\cos\theta)^2 + (-8\sin\theta)^2}\,d\theta$$

$$= 2\int_0^{\pi} \sqrt{64+128\cos\theta+64\cos^2\theta+64\sin^2\theta}\,d\theta$$

$$= 2\int_0^{\pi} \sqrt{64+128\cos\theta+64}\,d\theta$$

$$= 16\sqrt{2}\int_0^{\pi} \sqrt{1+\cos\theta}\,d\theta$$

$$= 16\sqrt{2}\int_0^{\pi} \frac{\sqrt{1-\cos^2\theta}}{\sqrt{1-\cos\theta}}\,d\theta$$

$$= 16\sqrt{2}\int_0^{\pi} \frac{\sin\theta}{\sqrt{1-\cos\theta}}\,d\theta$$

$$= 16\sqrt{2}\int_0^{2} u^{-\frac{1}{2}}\,du$$

$$= 32\sqrt{2}u^{\frac{1}{2}}\Big|_0^2 = 64$$

OR

$$= 16\sqrt{2}\int_0^{\pi} \sqrt{2\cos^2\left(\frac{\theta}{2}\right)}\,d\theta$$

$$= 32\int_0^{\pi} \cos\left(\frac{\theta}{2}\right)d\theta$$

$$= 64\sin\left(\frac{\theta}{2}\right)\Big|_0^{\pi} = 64$$

What have we learned?

- Can I find the arc length of a region on a polar graph?

April 20, 2016