WARMUP!!

Problem #4 on:

Solution:

10.4 - Polar Equations with Calculus!

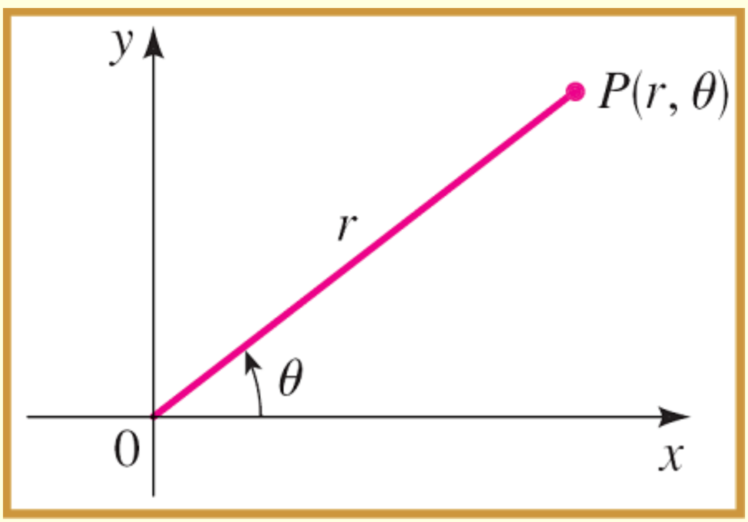
Essential Learning Targets:

- I can extend methods of calculating derivatives of real-valued functions to vector-valued functions, parametric functions, and functions in polar coordinates.
- I can use derivatives of r, x, and y with respect to θ and first and second derivatives of y with respect to x to provide information about a curve given by a polar equation $r = f(\theta)$.

A polar coordinate describes the location of the point in terms of direction and distance

The location of a point is given by an ordered pair (r, θ).

- **r is the distance from the origin (or pole).**
- **θ is the angle from the positive x-axis.**

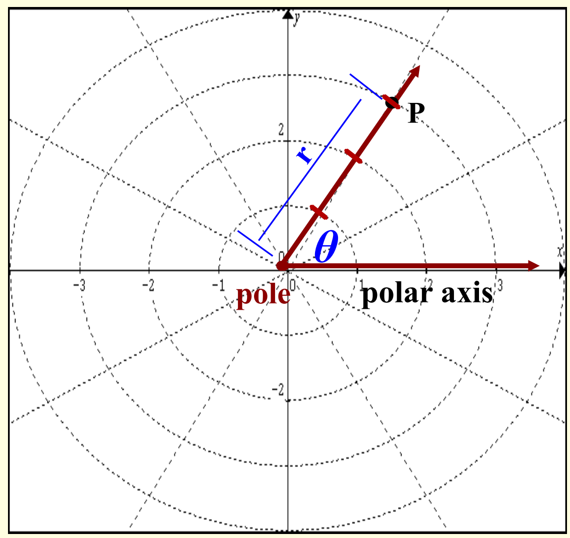


The Polar Axis

Polar coordinates are where: P(*r*, *θ*)

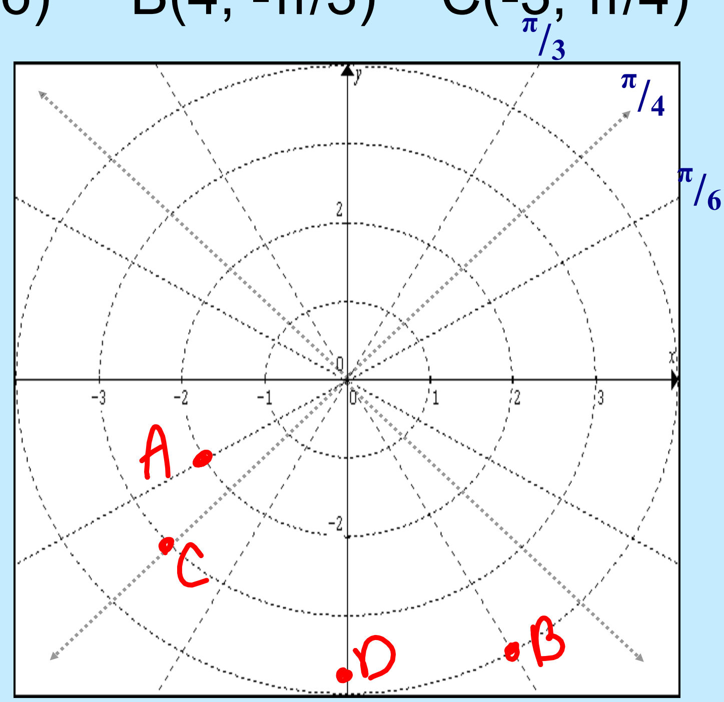
- **r is the radius**
  the distance from P to the pole(origin) is | r |
- **r can be positive or negative**

  if *r* is negative, then P is on the ray *opposite* from the terminal ray of *θ*
- ***θ* is the angle**
  ***θ* is rotational**
  (can be positive or negative)



ex) Plot the following points on the polar coordinate plane below:

A(2, 7π/6) B(4, -π/3) C(-3, π/4) D(-4, -3π/2)



To convert from rectangular coordinates to polar coordinates, remember that $x^2 + y^2 = r^2$ and $\tan\theta = y/x$

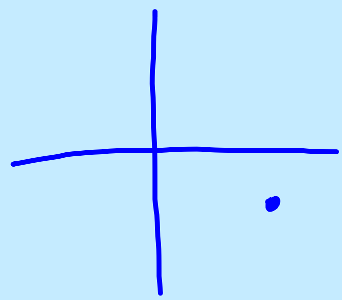
ex) Convert $(3, -\sqrt{3})$ from rectangular to polar coordinates if $r > 0$ and θ is in $[0, 2\pi)$

$$r^2 = 3^2 + (-\sqrt{3})^2 = 12$$

$$r = \pm\sqrt{12} = \pm 2\sqrt{3}$$

$$\tan\theta = \frac{-\sqrt{3}}{3}$$

$$\theta = \frac{5\pi}{6}, \frac{11\pi}{6} \text{ or } ~~-\frac{\pi}{6}~~$$



$$\boxed{\left(2\sqrt{3}, \frac{11\pi}{6}\right)} \quad ~~\left(-2\sqrt{3}, \frac{5\pi}{6}\right)~~$$

To convert from polar coordinates to rectangular coordinates, remember that

$x = r\cos\theta$ and $y = r\sin\theta$

ex) Convert (-3, 5π/3) to rectangular coordinates

(r = -3, θ = 5π/3)

$x = -3\cos\frac{5\pi}{3} = -\frac{3}{2}$

$y = -3\sin\frac{5\pi}{3}$

$= \frac{3\sqrt{3}}{2}$

$\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

Brain Break: It's getting to know you Tuesday!

DERIVATIVES OF POLAR EQUATIONS

Using: $x = r\cos\theta$ and $y = r\sin\theta$

dy/dx is just like parametric in that

$dy/dx = (dy/d\theta)/(dx/d\theta)$

ex) Suppose $r = \sin\theta + 2$. Find dy/dx.

FIND Y: $y = r\sin\theta = \sin^2\theta + 2\sin\theta$

FIND X: $x = r\cos\theta = \sin\theta\cos\theta + 2\cos\theta$

so $dy/d\theta = 2\sin\theta\cos\theta + 2\cos\theta$

so $dx/d\theta = \cos^2\theta - \sin^2\theta - 2\sin\theta$

$$dy/dx = \frac{2\sin\theta\cos\theta + 2\cos\theta}{\cos^2\theta - \sin^2\theta - 2\sin\theta}$$

Find dy/dx and the slopes of the lines tangent to the curve at (3, 7π/6) and (4, 3π/2) if r = 2(1 - sinθ)

$$x = r\cos\theta = 2(1-\sin\theta)\cos\theta = 2\cos\theta - 2\sin\theta\cos\theta$$

$$y = r\sin\theta = 2(1-\sin\theta)\sin\theta = 2\sin\theta - 2\sin^2\theta$$

$$\frac{dy}{d\theta} = 2\cos\theta - 4\sin\theta\cos\theta$$

$$\frac{dx}{d\theta} = -2\sin\theta - 2\cos^2\theta + 2\sin^2\theta$$

$$\frac{dy}{dx} = \frac{2\cos\theta - 4\sin\theta\cos\theta}{-2\sin\theta - 2\cos^2\theta + 2\sin^2\theta}$$

Slope at (3, 7π/6):

$$\frac{2\left(-\frac{\sqrt{3}}{2}\right) - 4\left(-\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)}{-2\left(-\frac{1}{2}\right) - 2\left(\frac{3}{4}\right) + 2\left(\frac{1}{4}\right)} = \frac{-\sqrt{3}-\sqrt{3}}{1-\frac{3}{2}+\frac{1}{2}} = undefined$$

Slope at (4, 3π/2)

$$\frac{2(0) - 4(-1)(0)}{-2(-1) - 2(0) + 2(1)} = \frac{0}{4} = 0$$

ex) Find the points of horizontal tangency to the polar curve r = 2 + 3sinθ for 0 ≤ θ < 2π

$$x = r\cos\theta = (2 + 3\sin\theta)\cos\theta = 2\cos\theta + 3\sin\theta\cos\theta$$

$$y = r\sin\theta = (2 + 3\sin\theta)\sin\theta = 2\sin\theta + 3\sin^2\theta$$

$$\frac{dy}{d\theta} = 2\cos\theta + 6\sin\theta\cos\theta$$

$$\frac{dx}{d\theta} = -2\sin\theta + 3\cos^2\theta - 3\sin^2\theta$$

$$\frac{dy}{dx} = \frac{2\cos\theta + 6\sin\theta\cos\theta}{-2\sin\theta + 3\cos^2\theta - 3\sin^2\theta}$$

Horizontal tangent occurs when $\frac{dy}{d\theta} = 0$

$$2\cos\theta + 6\sin\theta\cos\theta = 0$$

$$2\cos\theta\,(1 + 3\sin\theta) = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad or$$

$$\theta = \arcsin\left(-\frac{1}{3}\right) \approx 3.481, 5.943 \;\text{ on } [0, 2\pi]$$

so the points (in polar form) are:

$$\left(5, \frac{\pi}{2}\right), \left(-1, \frac{3\pi}{2}\right), (1, 3.481), (1, 5.943)$$

What have we learned?

- Can I find the derivative of polar equations?