WARMUP!!

Group work) Determine any differences between the curves of the parametric equations below. Are the graphs the same? Are the curves smooth? How can you check for this?

a) $x = 2\cos\theta,\ y = 2\sin\theta$

b) $x = \sqrt{t},\ y = \sqrt{4 - t}$

First set:

$$\cos\theta = \frac{x}{2} \quad and \quad \sin\theta = \frac{y}{2}$$

$$so \left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 1 \quad and\ x^2 + y^2 = 4$$

Second set:

$$t = x^2, so\ y = \sqrt{4 - x^2} \quad and \quad x^2 + y^2 = 4$$

So the equations match. But are they truly the same graphs? Check the domains.

The domain/range of the first set is [-2, 2] for both x and y.

The domain/range of the second set is [0, 2] for both x and y.

This means that the graph of the first set is the entire circle, while the graph of the second set is just the quarter in the first quadrant.

Let's check the orientations.

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
x	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	$-\sqrt{2}$	0	$\sqrt{2}$	2
y	0	$\sqrt{2}$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	$-\sqrt{2}$	0

t	0	1	2	3	4
x	0	1	$\sqrt{2}$	$\sqrt{3}$	2
y	2	$\sqrt{3}$	$\sqrt{2}$	1	0

So the first graph travels counter-clockwise while the second travels clockwise.

Smooth?

Both curves are smooth because their derivatives are always defined (with the exception of the endpoints on the 2nd set).

10.2b - Review of Parametric Equations

At the end of this lesson the students will be able to:

- find a set of parametric equations to represent a curve
- use a calculator to graph parametric equations

Try this! Find the standard form of the rectangular equation below. (Hint: try eliminating the parameter.)

Hyperbola: $x = h + a\sec\theta$, $y = k + b\tan\theta$

$$\sec\theta = \frac{x-h}{a}, \qquad \tan\theta = \frac{y-k}{b}$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Can you do a similar problem backwards?

Suppose the vertices of a hyperbola are (0, ±1) and the foci are (0, ±2). Write the equation of the hyperbola in standard form. Then write a set of parametric equations for the hyperbola.

Since the vertices are (0, ±1), we know that the center is at (0, 0).

We also know that the 'y' will come first because the hyperbola is vertically oriented.

Since $a^2 + b^2 = c^2$, we get that $1 + b^2 = 4$ so $b = \pm\sqrt{3}$.

$$\frac{y^2}{1} - \frac{x^2}{3} = 1$$

We know that $\sec^2\theta - \tan^2\theta = 1$

So $y^2 = \sec^2\theta$ and $x^2 = 3\tan^2\theta$.

$x = \sqrt{3}\tan\theta$ and $y = \sec\theta$

ex) Find 2 different sets of parametric equations for the rectangular equation $y = x^2$.

The easiest way to accomplish this is to just let either $x = t$ or $y = t$. However, you should know that you can create any relationship between x or y and t that you want. So you can let $x = t^4$ or $y = t^2 - 5$ or whatever floats your boat. The only requirement for this is that the relationship between x and y is maintained.

I let $x = t$ to get: $x = t$, $y = t^2$

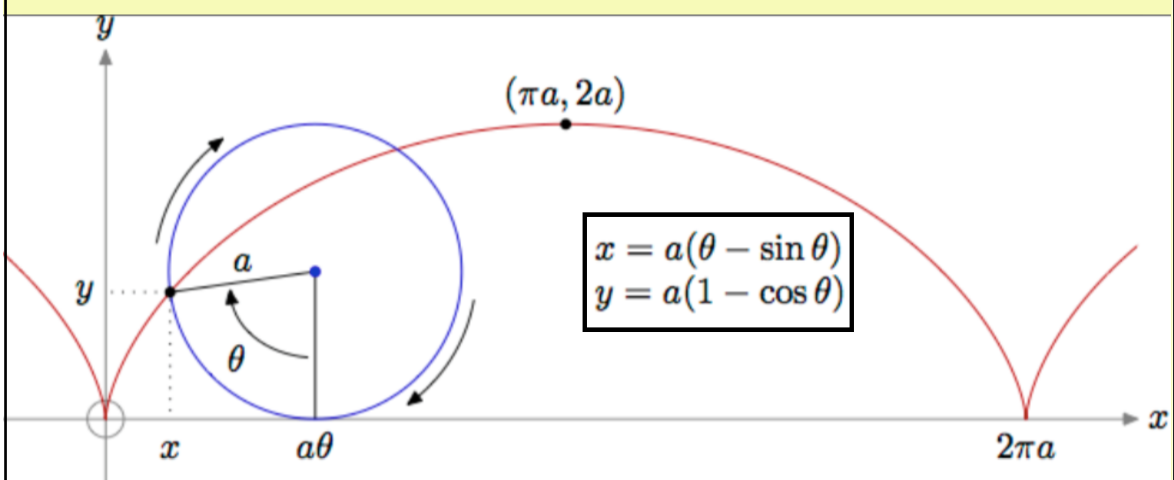
I let $y = t$ to get: $x = \sqrt{t}$, $y = t$

ex) Determine the curve traced by a point P on the circumference of a circle of radius a rolling along a straight line in a plane. (This curve is called a <u>cycloid</u>).

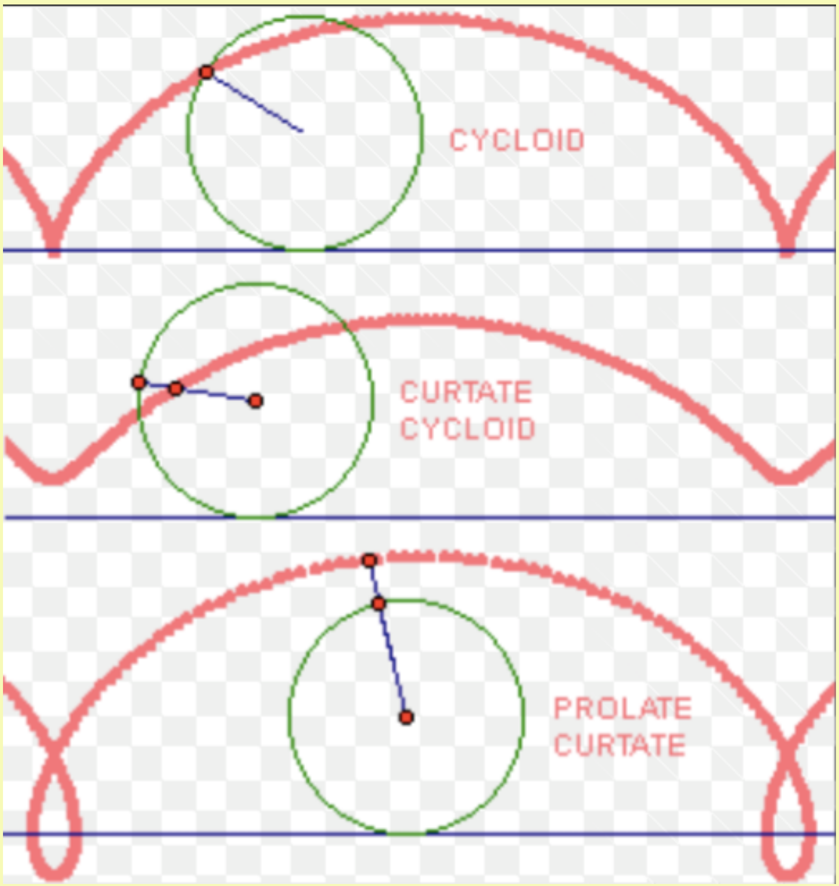
What does this look like?

Let P(x, y) begin at the origin. Let θ be the angle of the circle's rotation. Let α be the radius of the circle. So for one complete rotation, you will move a distance of 2πα along the x-axis. The maximum height would be the diameter of the circle, which occurs at the halfway point which would be at P(πα, 2α), when θ = π.

Because the angle is oriented sideways, αsinθ actually represents the horizontal component from the center to the circle to P and αcosθ represents the vertical component. Since αθ is the horizontal distance from the origin to the center of the circle, the distance from the origin to the x-value of P would be αθ - αsinθ. The y is similar. So we end up with:



There are different types of cycloids!



and others!! Wow!

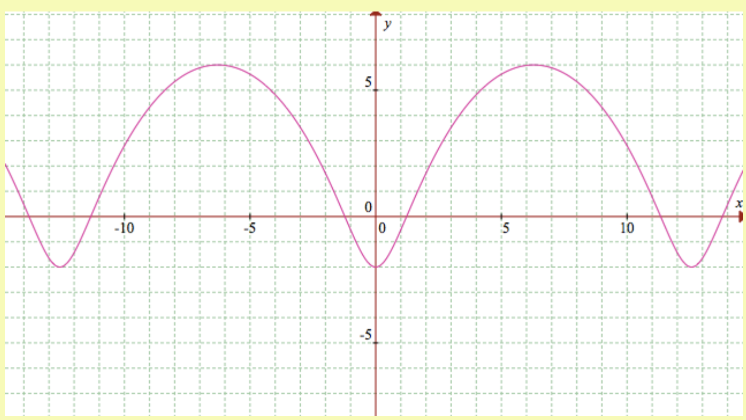
ex) Use your calculator to graph the curve of the curtate cycloid represented by x = 2θ - sinθ and y = 2 - 4cosθ. Indicate the direction and identify any points at which the curve is not smooth.

Make sure your mode is set to 'parametric' and 'radians'. Enter the equations in but make sure to check your window. Start with a window as follows:

tmin = -15	xmin = -15	ymin = -3
tmax = 15	xmax = 15	ymax = 8
tstep = 1	xstep = 1	ystep = 1

Looks a little choppy right?! How do we make it appear smoother? Make the tstep smaller. Try a tstep of 0.1 and see what happens.

The t-step affects the t-values that are used to create the set of points being graphed. The smaller the step, the smoother the appearance of the curve. (Don't go too small though, because it will take forever to graph.)



The derivatives of both equations are always defined so the curve is smooth. :)

Let's do some applications!!

What have we learned?

- Can I find a set of parametric equations to represent a curve?
- Can I use a calculator to graph parametric equations?