# Warmup!!

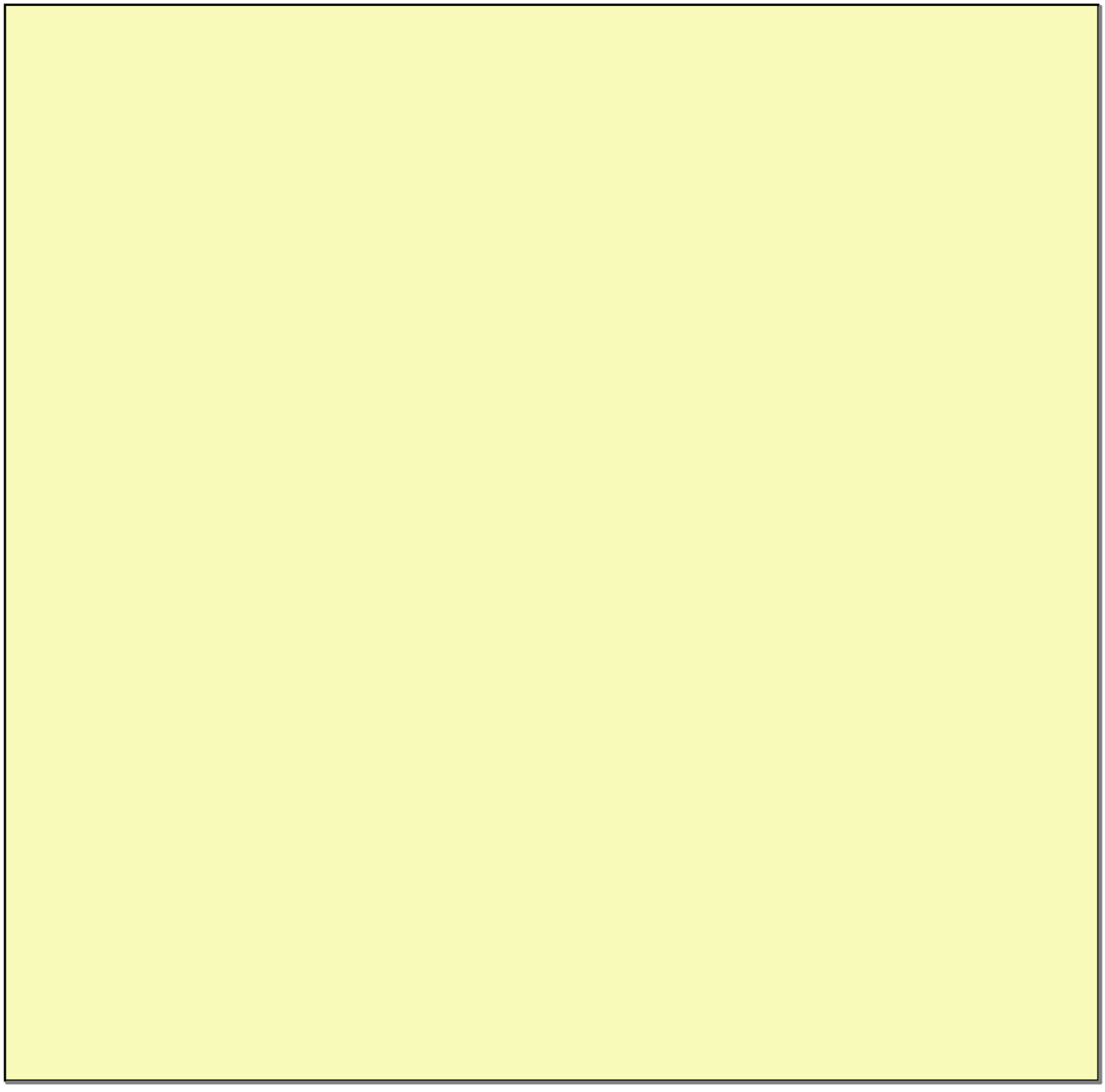
Let's see what you can do!

Given the parametric equations: $x = 2t^2$ and $y = t^4 + 1$, rewrite these as one rectangular equation (an equation in terms of x and y only).

Solving the first equation for $t^2$, we get $t^2 = x/2$
Substituting this into the 2nd equation, we get
$y = (t^2)^2 + 1 = (x/2)^2 + 1$

So $y = 1/4\ x^4 + 1$

April 11, 2016



$x = t^2$

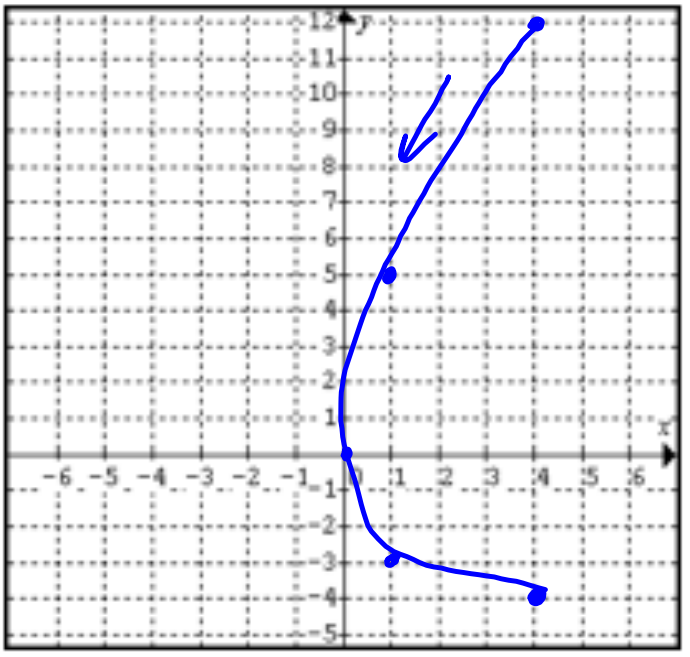
$y = t^2 - 4t$

$-2 \le t \le 2$

Construct a table with $t$, $x$, and $y$.

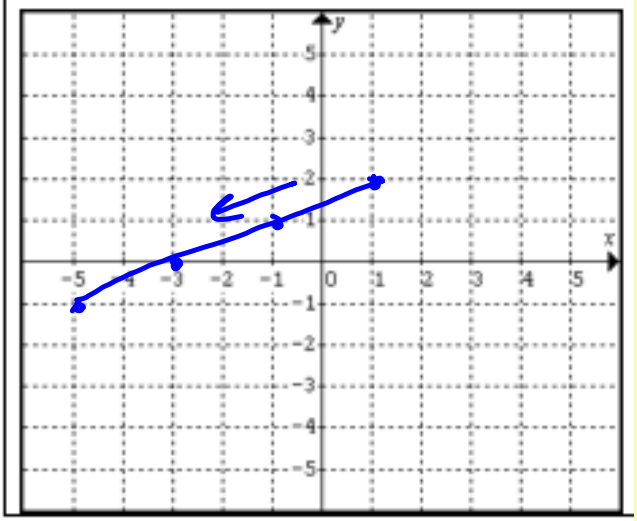
| $t$ | $x$ | $y$ |
|---|---|---|
| -2 | 4 | 12 |
| -1 | 1 | 5 |
| 0 | 0 | 0 |
| 1 | 1 | -3 |
| 2 | 4 | -4 |

Graph the $x$-$y$ coordinates and show the direction.



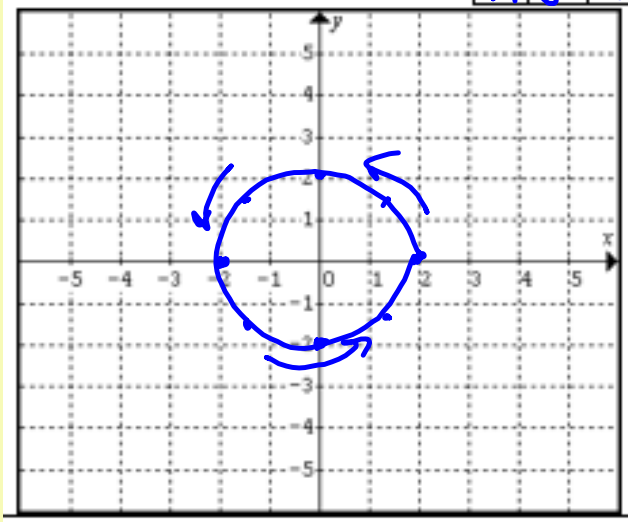
2. $x = 1 - 2t$, $y = 2 - t$, $0 \le t \le 3$

| $t$ | $x$ | $y$ |
|---|---|---|
| 0 | 1 | 2 |
| 1 | -1 | 1 |
| 2 | -3 | 0 |
| 3 | -5 | -1 |
| | | |



3. $x = 2\cos\theta$, $y = 2\sin\theta$, $0 \le \theta \le 2\pi$

| $\theta$ | $x$ | $y$ |
|---|---|---|
| 0 | 2 | 0 |
| $\pi/4$ | $\sqrt{2}$ | $\sqrt{2}$ |
| $\pi/2$ | 0 | 2 |
| $3\pi/4$ | $-\sqrt{2}$ | $\sqrt{2}$ |
| $\pi$ | -2 | 0 |
| $5\pi/4$ | $-\sqrt{2}$ | $-\sqrt{2}$ |
| $3\pi/2$ | 0 | -2 |
| $7\pi/4$ | $\sqrt{2}$ | $-\sqrt{2}$ |



*Example 4:*

**Eliminate the parameter and write the rectangular equation.**

$x = 1 - 2t$

$y = 2 - t$

$y = \frac{1}{2}x + \frac{3}{2}$

**Find the domain.** We must limit the $x$-values (domain) to obtain *exactly* the same graph as before. So, using the original parameter, $0 \le t \le 3$, and the equation that relates $x$ and $t$, **algebraically** find the domain for this graph. 1 -5

$D: [-5, 1]$

**Find the range.** Sometimes just the "domain" can be misleading, so you can do the same thing with the $y$-values. Again, use the original parameter, $0 \le t \le 3$, and the equation that relates $y$ and $t$ to **algebraically** find the range for this graph. 2 -1

$R: [-1, 2]$

5. $x = t^2 - 4$, $y = \frac{t}{2}$ $\quad -2 \le t \le 3$

Eliminate the parameter and write the rectangular equation.

$t = 2y$

$x = (2y)^2 - 4$

$x = 4y^2 - 4$

$y^2 = \frac{1}{4}x + 1$ $\quad y = \pm\sqrt{\frac{1}{4}x + 1}$

Algebraically, find the domain.

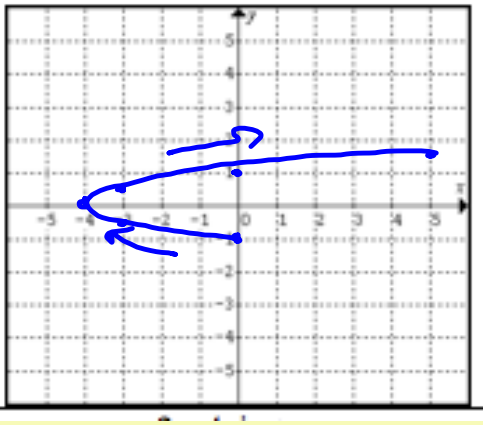
$x = 0$ to 5 hmmm

at $t = 0, x = -4$

so D: $[-4, 5]$

Confirm your answers by graphing.

| $t$ | $x$ | $y$ |
|---|---|---|
| -2 | 0 | -1 |
| -1 | -3 | $-\frac{1}{2}$ |
| 0 | -4 | 0 |
| 1 | -3 | $\frac{1}{2}$ |
| 2 | 0 | 1 |
| 3 | 5 | $\frac{3}{2}$ |



6. $x = 2\cos\theta$, $y = 2\sin\theta$ $\quad 0 \le \theta \le 2\pi$

Eliminate the parameter and write the rectangular equation.

Hint: Square both sides of each equation, add the equations together, and look for a trigonometric identity that you can replace!

$x^2 = 4\cos^2\theta$

$y^2 = 4\sin^2\theta$

$x^2 + y^2 = 4\cos^2\theta + 4\sin^2\theta$

$x^2 + y^2 = 4(1)$ $\quad x^2 + y^2 = 4$

Algebraically, find the domain.

Note: Because of the periodic nature of the sin and cos functions, when **algebraically** solving for the **domain**, you must use half the parameter!

7. $x = -2 + 4\sin t$, $y = 7 + 3\cos t$ $\quad 0 \le t \le 2\pi$

Eliminate the parameter and write the rectangular equation.

$4\sin t = x + 2$ $\quad 16\sin^2 t = (x+2)^2$

$3\cos t = y - 7$ $\quad 9\cos^2 t = (y-7)^2$

$\sin^2 t + \cos^2 t = \frac{(x+2)^2}{16} + \frac{(y-7)^2}{9}$

$\frac{(x+2)^2}{16} + \frac{(y-7)^2}{9} = 1$

Think about what this equation would graph and tell the domain – do not solve algebraically.

center: $(-2, 7)$

D: $[-6, 2]$

# 10.2a - Parametric Equations!

At the end of this lesson the students will be able to:

- sketch the graph of a curve with a given set of parametric equations
- eliminate the parameter in a set of parametric equations

# What have we learned?

- Can I sketch the graph of a curve with a given set of parametric equations?
- Can I eliminate the parameter in a set of parametric equations?