WARMUP!!

Ex) Find all vertical asymptotes of the function $y = \dfrac{x^2 - 3x + 2}{x^2 - 4}$

$$y = \frac{(x-2)(x-1)}{(x+2)(x-2)}$$

There is a vertical asymptote at x = -2.

(There is a removable discontinuity (a hole) at x = 2.)

How do we know if there's a vertical asymptote?

For any rational function: $y = f(x) / g(x)$, if $g(c) = 0$ and $f(c) \neq 0$, then $x = c$ is a vertical asymptote. (In other words, if a factor on the bottom doesn't cancel with a factor on the top, there's an asymptote there.)

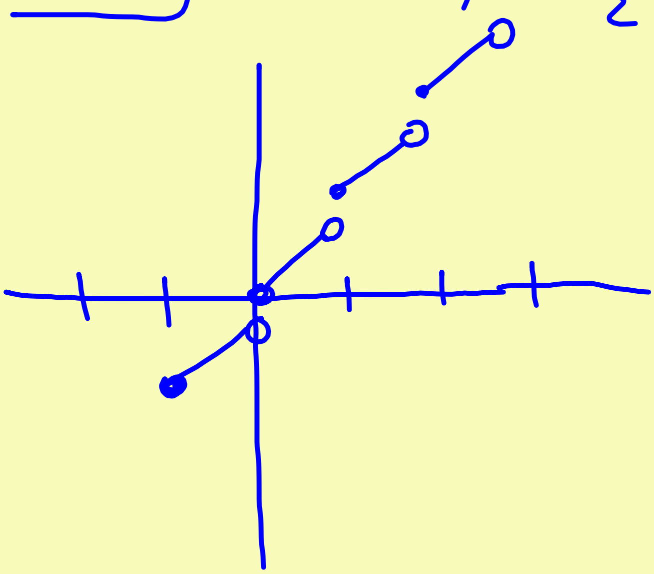
1-4

$\underline{19}$ $\lim\limits_{x \to \pi} \cot x$

$$= \lim_{x \to \pi} \frac{\cos x}{\sin x} = \frac{-1}{0}$$

$\Rightarrow$ DNE

$\underline{27}$ $f(x) = \frac{1}{2}[\![x]\!] + x$



$f(x)$ is discont at all integers b/c $\lim\limits_{x \to c} f(x)$ DNE

$c \in \mathbb{Z}$

41 $f(x) = \dfrac{x+2}{x^2-3x-10}$

$= \dfrac{x+2}{(x-5)(x+2)}$

f(x) is discont @ x = 5 and x = -2 b/c f(5) and f(-2) are und.

$\lim_{x \to -2} f(x) = -\frac{1}{7}$ so this discont. is removable

49

$$f(x) = \begin{cases} \tan\frac{\pi x}{4}, & |x| < 1 \\ x, & |x| \geq 1 \end{cases}$$

$$= \begin{cases} \tan\frac{\pi x}{4}, & -1 < x < 1 \\ x, & x \leq -1 \text{ or } x \geq 1 \end{cases}$$

51 $f(x) = \csc 2x = \dfrac{1}{\sin 2x}$

$\sin 2x = 0$

$2x = 0, \pi, 2\pi, \ldots$

$x = \ldots, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

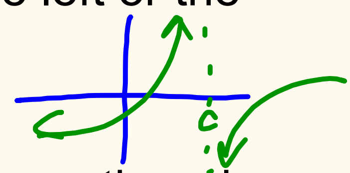
1.5 Infinite Limits!!

At the end of this lesson, you will be able to:

- Locate all vertical asymptotes for a function
- Determine the type of infinity (positive or negative) that a function is approaching on either side of a vertical asymptote

How do vertical asymptotes relate to limits?

If a vertical asymptote exists at x = c, this implies that the limit as x approaches c from either the left or the right approaches either ∞ or -∞.



Technically, these limits do not exist because they do not approach a real number, but traditionally we identify the infinity or negative infinity specifically to provide more information.

So if we say that the limit as x ⟶ c of f(x) = ∞, we are actually saying that this limit does not exist because the function increases without bound as x ⟶ c.

a) -5 b) infinity c) -5 d) DNE e) 1 f) 1 g) 4
h) 1 i) -2 j) -2 k) und l) -2

→ ∞

1 That's all great, but what does this mean for us??

plug in x

real #	$\frac{0}{0}$	not 0 / 0
☺	algebra	V.A.

Suppose we encounter the problem: $\lim_{x \to 2^-} \frac{x+1}{x-2} = \boxed{-\infty}$

plug in 1.9999

$\frac{+}{-}$

If we begin by plugging in, what do we get? NOT zero / zero

This means there's an asymptote there! So what do we do about it?

SHHHH!! This is the easy, cheating way to determine which infinity is the answer.

Pick an x-value 'close' to c, plug it in, and check the signs.

positive -> positive infinity ; negative -> negative infinity

$$\lim_{x \to 2^-} \frac{x+1}{x-2} = \boxed{-\infty}$$

$$\lim_{x \to 2^+} \frac{x+1}{x-2} = \boxed{\infty}$$

$\frac{+}{+}$

$$\lim_{x \to 2} \frac{x+1}{x-2} = \boxed{DNE}$$

Try these!

$a)\ \lim_{x\to 5^-} \frac{x^2-1000}{(x-5)^2}$ $= -\infty$

$b)\ \lim_{x\to 5^+} \frac{x^2-1000}{(x-5)^2}$ $= -\infty$

$c)\ \lim_{x\to 5} \frac{x^2-1000}{(x-5)^2}$ $= -\infty$

$d)\ \lim_{x\to -2^-} \frac{50-3x^2}{x^2-4}$ $= \infty$

$e)\ \lim_{x\to -2^+} \frac{50-3x^2}{x^2-4}$ $= -\infty$

f) What is my favorite food?

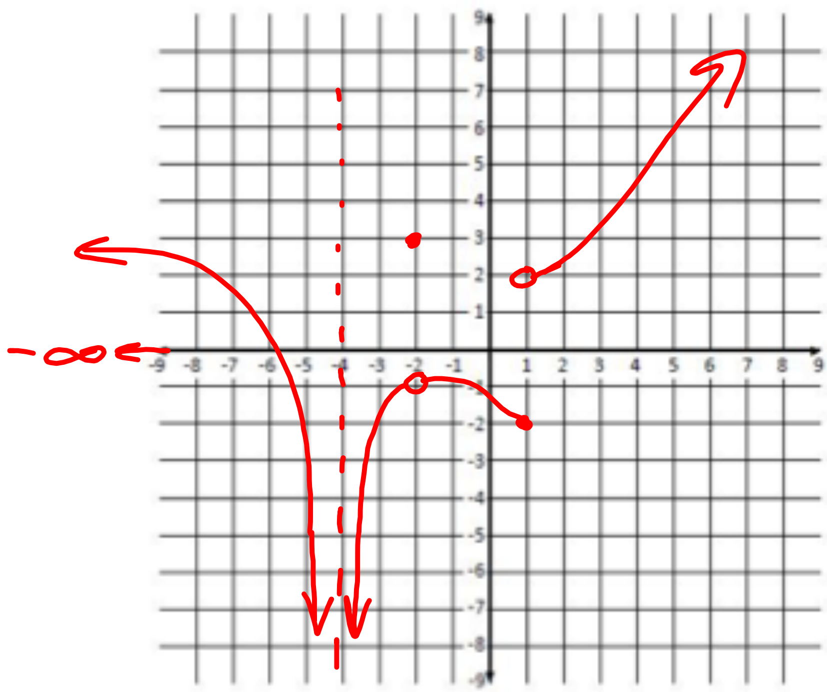
pizza

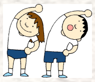
$f)\ \lim_{x\to -2} \frac{50-3x^2}{x^2-4}$ $=$ DNE

Let's take it up a notch!!

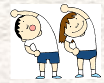
Sketch a possible graph for function $f(x)$ with the following properties:

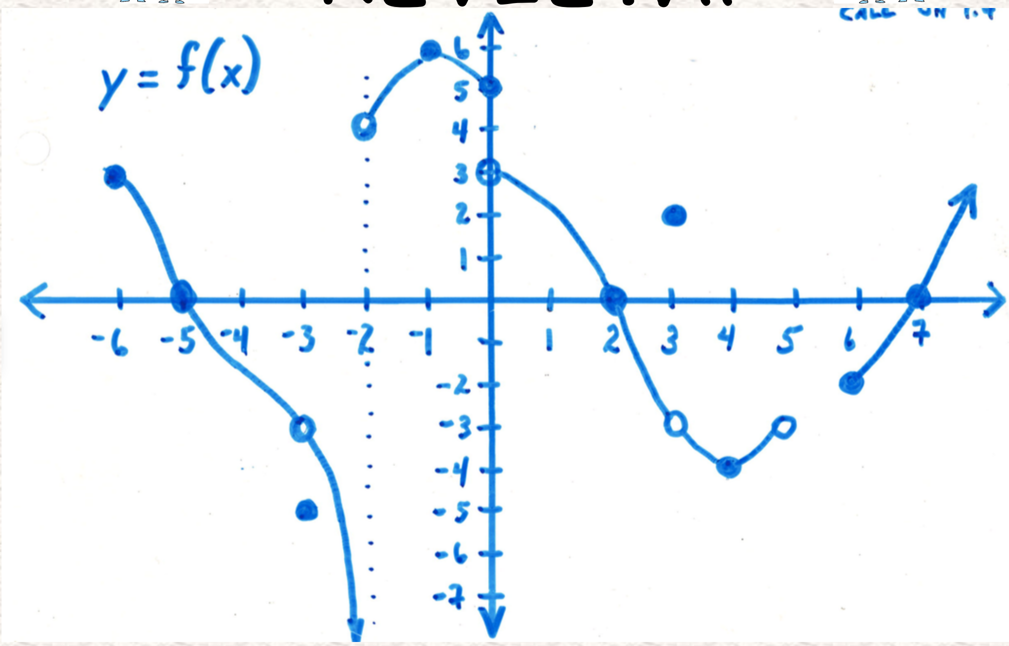
- $f(-2) = 3$
- $\lim_{x \to -2} f(x) = -1$
- $\lim_{x \to -4} f(x) = -\infty$
- $f(1) = -2$
- $\lim_{x \to 1^+} f(x) = 2$
- $\lim_{x \to -\infty} f(x) = 3$
- $\lim_{x \to \infty} f(x) = \infty$





REVIEW!!





1) Name all points of discontinuity over [-6, 4] and use the definition of continuity to state why the discontinuity exists.

2) Which of these discontinuities are removable?

3) Find:

a) f(3) b) f(5) c) f(-2) d) f(0)

e) $\lim_{x\to 2} f(x)$ f) $\lim_{x\to 6^-} f(x)$ g) $\lim_{x\to 0} f(x)$

h) $\lim_{x\to 0^+} f(x)$ i) $\lim_{x\to 0^-} f(x)$ j) $\lim_{x\to -3} f(x)$

✔ 1)

$x = -3$ because $\lim_{x\to -3} f(x) \neq f(-3)$

$x = -2$ because $f(-2)$ is undefined

$x = 0$ because $\lim_{x\to 0} f(x)$ *DNE*

$x = 3$ because $\lim_{x\to 3} f(x) \neq f(3)$

2) x = -3, 3

3) a) 2 b) und c) und d) 5 e) 0 f) DNE g) DNE h) 3 i) 5 j) -3

What have we learned??

1) What do we mean by left and right-handed limits?
2) What are the 3 conditions for continuity at a point?
3) What is an easy way to determine if a discontinuity is removable?