WARMUP!

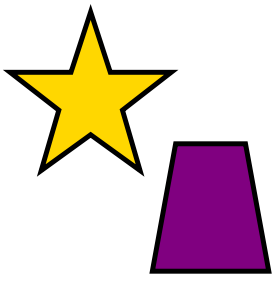
Without graphing or using a table, see if you can determine the value of each limit below.

1) $\lim_{x \to -1}(3x^5 - 2x^2 + 7x + 4) = -8$

$= 3(-1)^5 - 2(-1)^2 + 7(-1) + 4$

2) $\lim_{x \to 2} 5 = 5$

3) $\lim_{x \to \pi} x \cos x = -\pi$



Algebraic Limits

At the end of this lesson, you will be able to:

- find the value of a limit using direct substitution, factoring, rationalizing, simplifying fractions and simplifying using trigonometric identities
- use properties of limits to evaluate limits when a function is not given explicitly

Direct Substitution!

ALWAYS TRY THIS FIRST!!

For just about every continuous function (with just a few small exceptions), the limit is the same as the y-value associated with the given x-value.

$$\lim_{x \to 2}(x^2 + 3x)$$

$$= 2^2 + 3(2)$$
$$= 4 + 6$$
$$= 10$$

Properties of Limits:

Suppose the $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = K$, where b and c are constants. Then

- $\lim_{x \to c} b = b$
- $\lim_{x \to c} bf(x) = bL$
- $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
- $\lim_{x \to c} [f(x)g(x)] = LK$
- $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, given $K \neq 0$
- $\lim_{x \to c} f(x)^n = L^n$
- $\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right) = f(K)$

Indeterminate form of a limit:

When you substitute the x-value and the answer you initially get is 0/0, this is called an ***indeterminate form*** basically because you can't look at it and determine the form of the limit.

When you get this, don't despair, **keep going**!!!

98% of the time the limit exists!

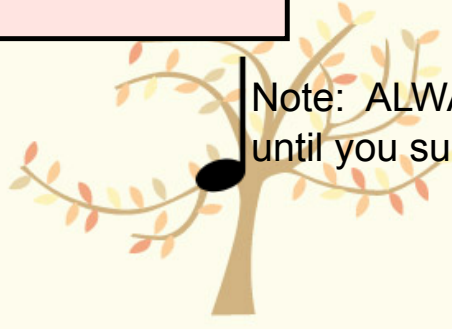
Remember, 0/0 means **keep going**!!!

You said keep going, but what do we do??

Method 1: everybody's favorite f-word...

factoring!!

$$ex)\ \lim_{x\to 1}\frac{x^2-6x+5}{2x^2-5x+3}=\lim_{x\to 1}\frac{(x-5)(x-1)}{(2x-3)(x-1)}=\lim_{x\to 1}\frac{x-5}{2x-3}=\frac{-4}{-1}=4$$



Note: ALWAYS rewrite the limit statement for EVERY step until you substitute in the x-value

Your turn!! Find the following:

$$\lim_{x\to -2}\frac{x^2+5x+6}{x+2} = 1$$

$$\lim_{x\to 2}\frac{(x+2)(x+3)}{x+2} = 0 = 1$$

$$\lim_{x\to 1}\frac{x^2+5x+6}{x-1} = \text{DNE}$$

$$\frac{12}{0}$$

so the limit DNE

$$\lim_{x\to 2}\frac{x^2-4}{x+2} = \frac{0}{4} = 0$$

$$\lim_{x\to 3}\frac{x^2-4}{x+2} = \frac{9-4}{3+2} = 1$$

What if you KNOW it factors but you just can't figure out how to factor it? Try division!

(How do you know this? If you have a rational function and the limit yields 0/0, it factors!)

$$\lim_{x\to -3} \frac{3x^3+13x^2+11x-3}{x^2+5x+6}$$

$$\begin{array}{r|rrrr} -3 & 3 & 13 & 11 & -3 \\ & & -9 & -12 & 3 \\ \hline & 3 & 4 & -1 & 0 \end{array}$$

$$= \lim_{x\to -3} \frac{(x+3)(3x^2+4x-1)}{(x+2)(x+3)}$$

$$= \frac{3(9)+4(-3)-1}{-3+2} = \frac{14}{-1} = -14$$

Take it up a notch!

When you see a radical, try rationalizing!

$$\lim_{x\to 4}\frac{6-\sqrt{x+32}}{x-4}\cdot\frac{6+\sqrt{x+32}}{6+\sqrt{x+32}}$$

$$=\lim_{x\to 4}\frac{36-(x+32)}{(x-4)(6+\sqrt{x+32})}$$

$$=\lim_{x\to 4}\frac{4-x}{(x-4)(6+\sqrt{x+32})}$$

$$=\lim_{x\to 4}\frac{-(x-4)}{(x-4)(6+\sqrt{x+32})}$$

$$=\frac{-1}{12}$$

Never distribute the terms in the denominator!

Take it up *another* notch!! It's time for trig!

3 important trig limit values:

$$1.\ \lim_{x\to 0}\frac{\sin x}{x}=1$$

$$2.\ \lim_{x\to 0}\frac{x}{\sin x}=1$$

$$3.\ \lim_{x\to 0}\frac{1-\cos x}{x}=0$$

$$\lim_{x\to 0}\frac{\sin(6x)}{6x}=1$$

Try these first. If they don't prove helpful then use identities to simplify.

Examples:

ex) $\lim_{x\to 0} \frac{\sin 5x}{x} =$ $\lim_{x\to 0} \frac{5}{5} \cdot \frac{\sin 5x}{x} = \lim_{x\to 0} 5 \cdot \frac{\sin 5x}{5x}$

$= 5 \cdot 1 = 5$

ex) $\lim_{x\to 0} \frac{3 - 3\cos x}{x} =$

$= \lim_{x\to 0} \frac{3(1-\cos x)}{x} = 3 \cdot 0 = 0$

ex) $\lim_{x\to 0} \frac{\sin^2 x}{x} =$

Remember: NEVER change the argument of a trig function without using an identity!!!

Your turn again!! Work these and check the answers with your group.

$$\lim_{x\to -5}\frac{2x^2+7x-15}{x+5} \checkmark = \lim_{x\to -5}\frac{(2x-3)(x+5)}{x+5} = \lim_{x\to -5}(2x-3) = -13$$

$$\lim_{x\to 1}\frac{5x^3-4x^2-3x+2}{x^2+3x-4}$$

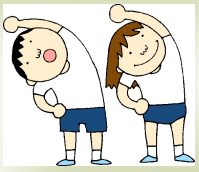
$$\checkmark = \lim_{x\to 1}\left[5x-19+74\frac{x-1}{(x+4)(x-1)}\right] = 5-19+\frac{74}{5} = \frac{4}{5}$$

$$\lim_{x\to 6}\frac{\sqrt{10+x}-4}{x-6} = \lim_{x\to 6}\frac{10+x-16}{(x-6)\left(\sqrt{10+x}+4\right)} = \lim_{x\to 6}\frac{x-6}{(x-6)\left(\sqrt{10+x}+4\right)}$$

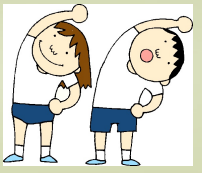
$$\checkmark = \lim_{x\to 6}\frac{1}{\sqrt{10+x}+4} = \frac{1}{8}$$

$$\lim_{x\to 0}\frac{\frac{1}{5x+2}-\frac{1}{2}}{x}$$

$$\checkmark = \lim_{x\to 0}\frac{2-(5x+2)}{2x(5x+2)} = \lim_{x\to 0}\frac{-5x}{2x(5x+2)} = \lim_{x\to 0}\frac{-5}{2(5x+2)} = -\frac{5}{4}$$



REVIEW!!



Find the slopes of the lines through each pair of points

1. (2, 1) and (-5, 7)
2. (a, b) and (c, d)
3. f(x) = 2x - 5; (3, f(3)) and (5, f(5))
4. f(x) = 2x -5; (a, f(a)) and (a + h, f(a + h))
5. f(x) = x^2; (a, f(a)) and (a + h, f(a + h))

✔ **1. -6/7 2. (d - b)/(c - a) OR (b - d)/(a - c) 3. 2 4. 2 5. 2a + h**

What have we learned??

1) What are some of the methods I can use to find limits algebraically?
2) What should I always do first when finding a limit?
3) What are the 3 trig limit values I should know?
4) What is something I should NEVER do with trig functions?